This problem is designed to walk you through the mechanics of the Solow growth model with population growth and technological progress

1. Suppose the production function is $Y = 10K^{\alpha}(EL)^{1-\alpha}$ and capital lasts an average of 10 years. Assume the population grows at a rate of 4%, labor earns 75% of total output, and that the rate of technological progress is 2%. The marginal product of capital is $MPK = 2.5\tilde{k}^{\alpha-1}$, where $\tilde{k}$ is capital per effective worker. Households consume 88.8% of each additional unit of income.

    a. What is α equal to? How do you know?

    b. Write the production function in per effective worker terms, $f(\tilde{k})$.

    c. Using the Solow equation, compute the steady state capital stock (in per effective worker terms), $\tilde{k}^*$.

    d. Calculate the following values at their steady state levels: output per effective worker $\tilde{y}^*$, consumption per effective worker $\tilde{c}^*$, and investment per effective worker, $\tilde{\imath}^*$.

e. Illustrate the economy's steady state on a Solow diagram that includes production, actual investment (saving), and break-even investment. Indicate the numerical values you found from parts c) and d) above. Be sure to clearly label the axes.

f. Calculate the golden rule capital per effective worker $\tilde{k}_{gold}$ and the saving rate associated with the golden rule.

g. Calculate the following values at their golden rule levels: output per effective worker $\tilde{y}_{gold}$, consumption per effective worker $\tilde{c}_{gold}$, and investment per effective worker, $\tilde{\imath}_{gold}$.

h. Illustrate the golden rule values on your diagram from part e. Is capital above or below its golden rule value?

i. Suppose the government implements a change in tax policy that leads to a change in the savings rate that is consistent with the golden rule. Illustrate how output, consumption, and investment change in period one (when the savings rate changes), in the transition dynamics (moving from the initial to the new steady state), and at the new steady state.