This problem is designed to walk you through the mechanics of the Solow growth model with population growth (and no technological progress).

1. Suppose the amount of capital, K, remains constant over time, but the number of workers, L, grows at a rate of n = 2%. Compute the percentage change in the capital-worker ratio in this case. What happens to k each year?

2. Now, let's suppose the economy spends some of its resources on investment, and the depreciation rate is 5%. The change in the capital stock per worker will now be equal to:

$$\Delta k = i - \delta k - nk = i - 0.05k - 0.02k$$

   Recall that investment per worker is assumed to be a constant fraction, *s,* of output per worker (*i= sf(k)*). Assume the marginal propensity to consume is equal to 80%. Using this information, substitute out "i" in the expression above, so that the change in capital per worker is expresses as a function of capital per worker, k.

3. Assume that the production function is given by $Y = K^{1/2}L^{1/2}$. Rewrite the production function in per worker terms. What property of this production function allows you to rewrite the production function in this way?

4. Complete the table below, based on the values given above.

(1) Capital per worker *k*	(2) Output per worker *f(k)*	(3) Investment per worker *sf(k)*	(4) Break-even Investment per worker *(δ +n)k*	(5) Change in capital per worker
0				
4				
8				
16				
36				

5. Plot the production function (2), the savings per worker (5), and the depreciation per worker (3) using the values from your table above. According to your graph, what is the steady state capital stock? Label this Point A on your diagram.
6. Now, suppose the population growth rate decreases to 1.5% per year. What happens to the following variables over time, as the economy transitions to the new steady state: k, y, c. Illustrate this change on your diagram from #5 and then illustrate how *each* variable changes *over time*, using the space below.

This problem is designed to study the golden rule level of capital in a model with no population growth or technological progress.

1. One logical goal among policymakers might be to choose the steady state capital stock that maximizes consumption per worker, denoted k^*_{gold} . Although both the rate of depreciation, δ, and saving rate, *s*, are assumed to be exogenous, policymakers may be able to influence the savings rate. Provide two possible changes the legal system that would cause an increase in s.
2. As s increases, what happens to the steady-state capital per worker? What happens to the steady state *growth rate* of capital per worker?
3. Assume that the production function is given by $Y = K^{1/2}L^{1/2}$. Rewrite the production function in per worker terms. What property of this production function allows you to rewrite the production function in this way?
4. Assume that capital lasts an average of 20 years. What is the depreciation rate, δ?
5. Assume households consume 80% of their disposable income. What is the savings rate, *s*?
6. Using the information above, complete the table below. Identify the steady state capital per worker, k*.

(1) Capital per worker k	(2) Output per worker $f(k) = k^{1/2}$	(3) Depreciation per worker δk	(4) Consumption per worker $c = f(k) - \delta k$	(5) Saving per worker $sf(k) = sk^{1/2}$
0				
4				
16				
36				
64				
100				
121				
144				

7. Plot the production function (2), the savings per worker (5), and the depreciation per worker (3) using the values from your table above. According to your graph, what is the steady state capital stock? Label this Point A on your diagram.
8. Write out the steady state condition for the Solow model presented in today's lecture. Using algebra, find the steady state capital per worker, k*. Is this the same amount you found in #6 & #7?
9. Now, we can find the golden rule capital per worker.
    a. Plot the points for consumption per worker (4) on your graph.
    b. Find the golden rule capital stock. Label this Point B on your diagram.
    c. According to your graph, do policymakers need to increase or decrease the savings rate to achieve the golden rule?