CHAPTER 3

National Income: Where it Comes From and Where it Goes

In this chapter, you will learn…

- what determines the economy's total output/income
- how the prices of the factors of production are determined
- how total income is distributed
- what determines the demand for goods and services
- how equilibrium in the goods market is achieved

Outline of model

A closed economy, market-clearing model

Supply side
- factor markets (supply, demand, price)
- determination of output/income

Demand side
- determinants of ***C***, ***I***, and ***G***

Equilibrium
- goods market
- loanable funds market

Factors of production

K = capital:
tools, machines, and structures used in production

L = labor:
the physical and mental efforts of workers

The production function

- denoted $Y = F(K, L)$
- shows how much output (Y) the economy can produce from K units of capital and L units of labor
- reflects the economy's level of technology
- exhibits constant returns to scale

Returns to scale: A review

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor z:

$K_2 = zK_1$ and $L_2 = zL_1$

(*e.g.*, if $z = 1.25$, then all inputs are increased by 25%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If **constant returns to scale**, $Y_2 = zY_1$
- If **increasing returns to scale**, $Y_2 > zY_1$
- If **decreasing returns to scale**, $Y_2 < zY_1$

Example 1

$$F(K,L) = \sqrt{KL}$$

$$F(zK,zL) = \sqrt{(zK)(zL)}$$

$$= \sqrt{z^2KL}$$

$$= \sqrt{z^2}\sqrt{KL}$$

$$= z\sqrt{KL}$$

$$= zF(K,L)$$

constant returns to scale for any $z > 0$

Example 2

$$F(K,L) = \sqrt{K} + \sqrt{L}$$

$$F(zK,zL) = \sqrt{zK} + \sqrt{zL}$$

$$= \sqrt{z}\sqrt{K} + \sqrt{z}\sqrt{L}$$

$$= \sqrt{z}\left(\sqrt{K} + \sqrt{L}\right)$$

$$= \sqrt{z}\,F(K,L)$$

decreasing returns to scale for any $z > 1$

Example 3

$$F(K,L) = K^2 + L^2$$

$$F(zK,zL) = (zK)^2 + (zL)^2$$

$$= z^2\left(K^2 + L^2\right)$$

$$= z^2F(K,L)$$

increasing returns to scale for any $z > 1$

Now you try…

- Determine whether constant, decreasing, or increasing returns to scale for each of these production functions:

(a) $F(K,L) = \dfrac{K^2}{L}$

(b) $F(K,L) = K + L$

Answer to part (a)

$$F(K,L) = \frac{K^2}{L}$$

$$F(zK,zL) = \frac{(zK)^2}{zL}$$

$$= \frac{z^2K^2}{zL}$$

$$= z\frac{K^2}{L}$$

$$= zF(K,L)$$

constant returns to scale for any $z > 0$

Answer to part (b)

$$F(K,L) = K + L$$

$$F(zK,zL) = zK + zL$$

$$= z(K + L)$$

$$= zF(K,L)$$

constant returns to scale for any $z > 0$

Assumptions of the model

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at

$$K = \bar{K} \quad \text{and} \quad L = \bar{L}$$

Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\bar{Y} = F(\bar{K}, \bar{L})$$

The distribution of national income

- determined by **factor prices**, the prices per unit that firms pay for the factors of production
  - wage = price of ***L***
  - **rental rate** = price of ***K***

Notation

W	= nominal wage
R	= nominal rental rate
P	= price of output
W/P	= real wage (measured in units of output)
R/P	= real rental rate

How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?

Demand for labor

- Assume markets are competitive: each firm takes ***W***, ***R***, and ***P*** as given.
- Basic idea: A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

Marginal product of labor (*MPL*)

- definition:
  The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

$$MPL = F(K, L+1) - F(K, L)$$

Exercise: *Compute & graph MPL*

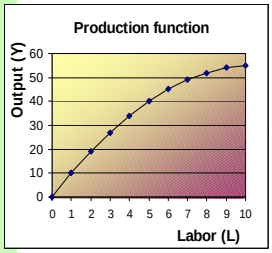
a. Determine ***MPL*** at each value of ***L***.

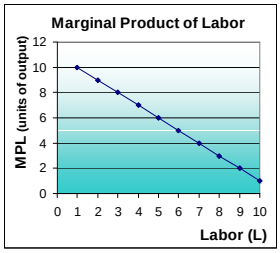
b. Graph the production function.

c. Graph the ***MPL*** curve with ***MPL*** on the vertical axis and ***L*** on the horizontal axis.

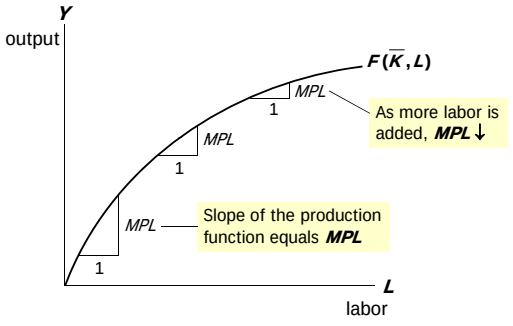
L	*Y*	*MPL*
0	0	n.a.
1	10	?
2	19	?
3	27	8
4	34	?
5	40	?
6	45	?
7	49	?
8	52	?
9	54	?
10	55	?

Answers:





MPL and the production function



Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).
- Intuition:
  Suppose ↑***L*** while holding ***K*** fixed
  ⇒ fewer machines per worker
  ⇒ lower worker productivity

Check your understanding:

- Which of these production functions have diminishing marginal returns to labor?

a) $F(K,L) = 2K + 15L$

b) $F(K,L) = \sqrt{KL}$

c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

Exercise *(part 2)*

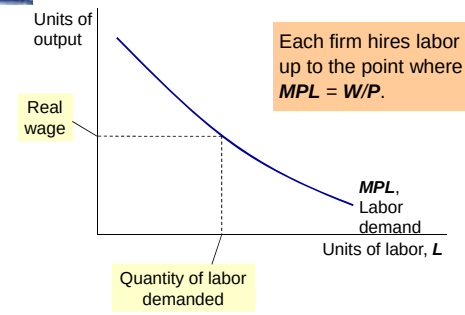
Suppose $W/P = 6$.

d. If $L = 3$, should firm hire more or less labor? Why?

e. If $L = 7$, should firm hire more or less labor? Why?

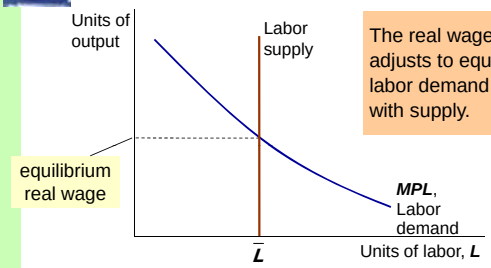
L	Y	MPL
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

MPL and the demand for labor



Each firm hires labor up to the point where $MPL = W/P$.

The equilibrium real wage



The real wage adjusts to equate labor demand with supply.

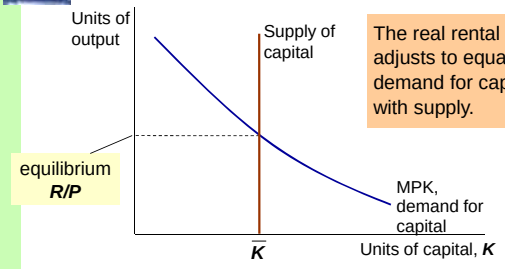
Determining the rental rate

We have just seen that $MPL = W/P$.

The same logic shows that $MPK = R/P$:

- diminishing returns to capital: $MPK \downarrow$ as $K \uparrow$
- The MPK curve is the firm's demand curve for renting capital.
- Firms maximize profits by choosing K such that $MPK = R/P$.

The equilibrium real rental rate



The real rental rate adjusts to equate demand for capital with supply.

The Neoclassical Theory of Distribution

- states that each factor input is paid its marginal product
- is accepted by most economists

How income is distributed:

total labor income = $\frac{W}{P}\bar{L} = MPL \times \bar{L}$

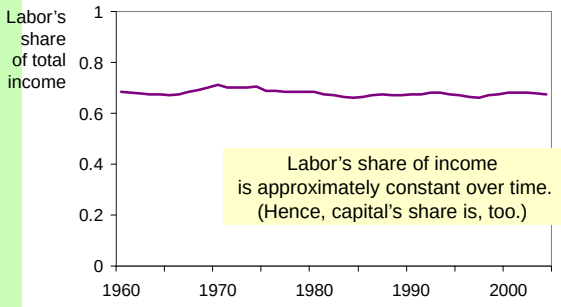
total capital income = $\frac{R}{P}\bar{K} = MPK \times \bar{K}$

If production function has constant returns to scale, then

$$\bar{Y} = \underbrace{MPL \times \bar{L}}_{\text{labor income}} + \underbrace{MPK \times \bar{K}}_{\text{capital income}}$$

national income

The ratio of labor income to total income in the U.S.



The Cobb-Douglas Production Function

- The Cobb-Douglas production function has constant factor shares:

  α = capital's share of total income:

  capital income = $MPK \times K = \alpha Y$

  labor income = $MPL \times L = (1-\alpha)Y$

- The Cobb-Douglas production function is:

$$Y = AK^{\alpha}L^{1-\alpha}$$

where A represents the level of technology.

The Cobb-Douglas Production Function

- Each factor's marginal product is proportional to its average product:

$$MPK = \alpha AK^{\alpha-1}L^{1-\alpha} = \frac{\alpha Y}{K}$$

$$MPL = (1-\alpha)AK^{\alpha}L^{-\alpha} = \frac{(1-\alpha)Y}{L}$$

Outline of model

A closed economy, market-clearing model

Supply side

DONE ✓ factor markets (supply, demand, price)

DONE ✓ determination of output/income

Demand side

Next → ❑ determinants of C, I, and G

Equilibrium

❑ goods market

❑ loanable funds market

Demand for goods & services

Components of aggregate demand:

C = consumer demand for g & s

I = demand for investment goods

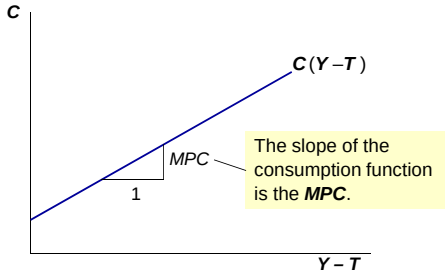
G = government demand for g & s

(closed economy: no NX)

Consumption, *C*

- def: **Disposable income** is total income minus total taxes: $Y - T$.
- Consumption function: $C = C(Y - T)$
  Shows that $\uparrow(Y - T) \Rightarrow \uparrow C$
- def: **Marginal propensity to consume (MPC)** is the increase in C caused by a one-unit increase in disposable income.

The consumption function

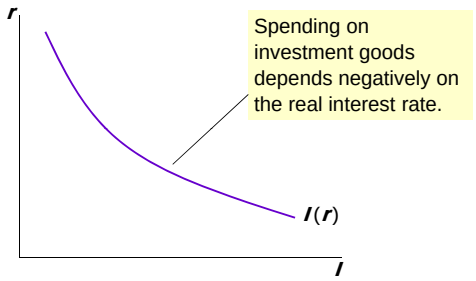


Investment, *I*

- The investment function is $I = I(r)$,
  where r denotes the **real interest rate**,
  the nominal interest rate corrected for inflation.
- The real interest rate is
  - the cost of borrowing
  - the opportunity cost of using one's own funds to finance investment spending.

So, $\uparrow r \Rightarrow \downarrow I$

The investment function



Government spending, *G*

- G = govt spending on goods and services.
- G excludes transfer payments
  (*e.g.*, social security benefits, unemployment insurance benefits).
- Assume government spending and total taxes are exogenous:

$$G = \bar{G} \quad \text{and} \quad T = \bar{T}$$

The market for goods & services

- Aggregate demand: $C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$
- Aggregate supply: $\bar{Y} = F(\bar{K}, \bar{L})$
- Equilibrium: $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$
- The real interest rate adjusts to equate demand with supply.

The loanable funds market

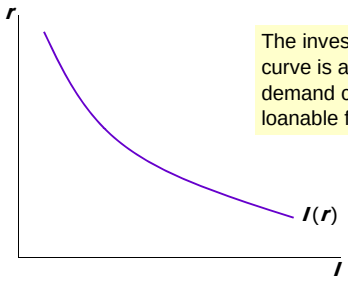
- A simple supply-demand model of the financial system.
- One asset: "loanable funds"
  - demand for funds: investment
  - supply of funds: saving
  - "price" of funds: real interest rate

Demand for funds: Investment

The demand for loanable funds…

- comes from investment:
  Firms borrow to finance spending on plant & equipment, new office buildings, etc. Consumers borrow to buy new houses.
- depends negatively on ***r***,
  the "price" of loanable funds
  (cost of borrowing).

Loanable funds demand curve



The investment curve is also the demand curve for loanable funds.

Supply of funds: Saving

- The supply of loanable funds comes from saving:
  - Households use their saving to make bank deposits, purchase bonds and other assets. These funds become available to firms to borrow to finance investment spending.
  - The government may also contribute to saving if it does not spend all the tax revenue it receives.

Types of saving

private saving $= (Y - T) - C$

public saving $= T - G$

national saving, S

= private saving + public saving

$= (Y - T) - C + T - G$

$= Y - C - G$

Notation: Δ = change in a variable

- For any variable X, ΔX = "the change in X"
  Δ is the Greek (uppercase) letter *Delta*

Examples:

- If $\Delta L = 1$ and $\Delta K = 0$, then $\Delta Y = MPL$.
  More generally, if $\Delta K = 0$, then $MPL = \frac{\Delta Y}{\Delta L}$.
- $\Delta(Y - T) = \Delta Y - \Delta T$, so
  $\Delta C = MPC \times (\Delta Y - \Delta T)$
  $= MPC\ \Delta Y - MPC\ \Delta T$

EXERCISE:

Calculate the change in saving

Suppose ***MPC*** = 0.8 and ***MPL*** = 20.

For each of the following, compute ΔS :

a. $\Delta G = 100$

b. $\Delta T = 100$

c. $\Delta Y = 100$

d. $\Delta L = 10$

Answers

$$\Delta S = \Delta Y - \Delta C - \Delta G = \Delta Y - 0.8(\Delta Y - \Delta T) - \Delta G$$
$$= 0.2\Delta Y + 0.8\Delta T - \Delta G$$

a. $\Delta S = -100$

b. $\Delta S = 0.8 \times 100 = 80$

c. $\Delta S = 0.2 \times 100 = 20$

d. $\Delta Y = \text{MPL} \times \Delta L = 20 \times 10 = 200,$

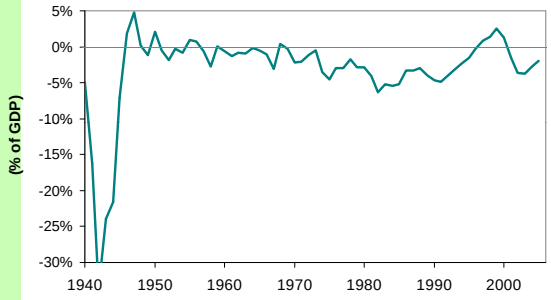
$\Delta S = 0.2 \times \Delta Y = 0.2 \times 200 = 40.$

digression:

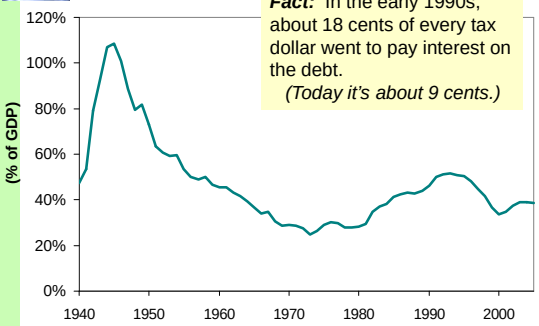
Budget surpluses and deficits

- If $T > G$, **budget surplus** $= (T - G)$ = public saving.
- If $T < G$, **budget deficit** $= (G - T)$ and public saving is negative.
- If $T = G$, "balanced budget," public saving = 0.
- The U.S. government finances its deficit by issuing Treasury bonds – *i.e.*, borrowing.

U.S. Federal Government Surplus/Deficit, 1940-2005



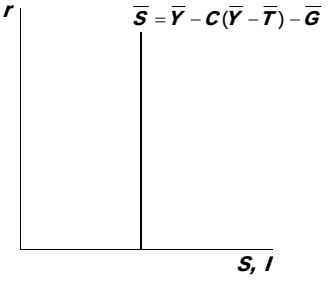
U.S. Federal Government Debt, 1940-2005



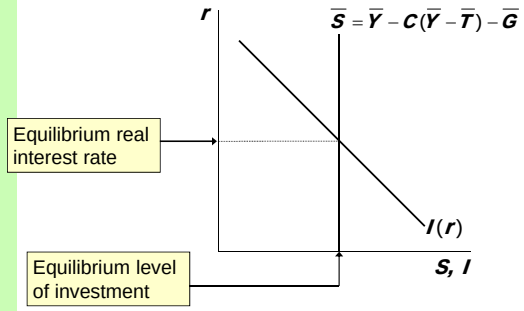
Fact: In the early 1990s, about 18 cents of every tax dollar went to pay interest on the debt.
(Today it's about 9 cents.)

Loanable funds supply curve

National saving does not depend on ***r***, so the supply curve is vertical.



Loanable funds market equilibrium



The special role of *r*

r adjusts to equilibrate the goods market and the loanable funds market simultaneously:

If L.F. market in equilibrium, then

$Y - C - G = I$

Add $(C+G)$ to both sides to get

$Y = C + I + G$ *(goods market eq'm)*

Thus,

Eq'm in L.F. market ⇔ Eq'm in goods market

Digression: Mastering models

To master a model, be sure to know:

1. Which of its variables are endogenous and which are exogenous.
2. For each curve in the diagram, know
   a. definition
   b. intuition for slope
   c. all the things that can shift the curve
3. Use the model to analyze the effects of each item in 2c.

Mastering the loanable funds model

Things that shift the saving curve

- public saving
  - fiscal policy: changes in *G* or *T*
- private saving
  - preferences
  - tax laws that affect saving
    - 401(k)
    - IRA
    - replace income tax with consumption tax

CASE STUDY: The Reagan deficits

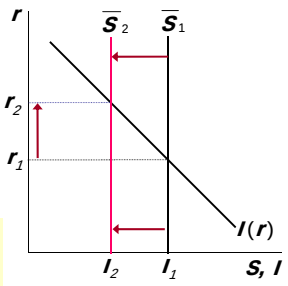
- Reagan policies during early 1980s:
  - increases in defense spending: $\Delta G > 0$
  - big tax cuts: $\Delta T < 0$
- Both policies reduce national saving:

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$

$\uparrow \bar{G} \Rightarrow \downarrow \bar{S}$ $\quad$ $\downarrow \bar{T} \Rightarrow \uparrow C \Rightarrow \downarrow \bar{S}$

CASE STUDY: The Reagan deficits

1. The increase in the deficit reduces saving…
2. …which causes the real interest rate to rise…
3. …which reduces the level of investment.



Are the data consistent with these results?

variable	*1970s*	*1980s*
T – G	–2.2	–3.9
S	19.6	17.4
r	1.1	6.3
I	19.9	19.4

T–G, S, and I are expressed as a percent of GDP
All figures are averages over the decade shown.

Now you try…

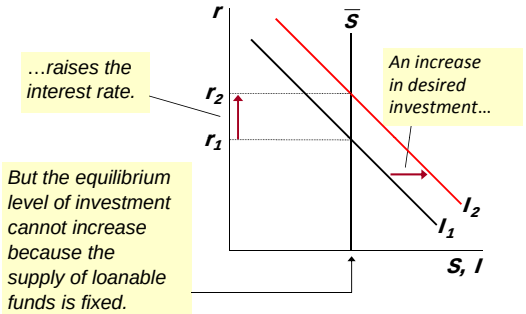
- Draw the diagram for the loanable funds model.
- Suppose the tax laws are altered to provide more incentives for private saving.
  (Assume that total tax revenue ***T*** does not change)
- What happens to the interest rate and investment?

Mastering the loanable funds model, *continued*

Things that shift the investment curve

- some technological innovations
  - to take advantage of the innovation, firms must buy new investment goods
- tax laws that affect investment
  - investment tax credit

An increase in investment demand

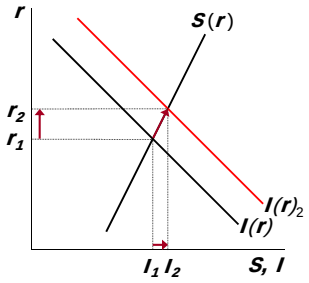


Saving and the interest rate

- Why might saving depend on ***r*** ?
- How would the results of an increase in investment demand be different?
  - Would ***r*** rise as much?
  - Would the equilibrium value of ***I*** change?

An increase in investment demand when saving depends on *r*

An increase in investment demand raises ***r***, which induces an increase in the quantity of saving, which allows ***I*** to increase.



Chapter Summary

- Total output is determined by
  - the economy's quantities of capital and labor
  - the level of technology
- Competitive firms hire each factor until its marginal product equals its price.
- If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

Chapter Summary

- A closed economy's output is used for
  - consumption
  - investment
  - government spending
- The real interest rate adjusts to equate the demand for and supply of
  - goods and services
  - loanable funds

Chapter Summary

- A decrease in national saving causes the interest rate to rise and investment to fall.
- An increase in investment demand causes the interest rate to rise, but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.