Astronomy 131: The Solar System and Space Exploration
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Supplementary Problem
The Sizes and Distances of the Moon and Sun

The intellectual advances of the ancient Greeks revolutionized astronomy. Thales (circa 600 B.C.) realized that the Moon shines by reflected sunlight. Followers of Pythagoras (circa 530 B.C.) believed that the Earth was round. Aristotle (circa 340 B.C.) also believed that the Earth was round because the Earth's shadow is circular when it passes across the Moon during a lunar eclipse. Aristarchus (circa 240 B.C.) took the idea of Heraclides (circa 340 BC) that the Earth spins on its axis, thereby explaining the motion of the stars across the sky on a daily basis. He also argued for a heliocentric, or Sun centered, model of the "universe". Aristarchus (a busy guy!) also devised a way to calculate the distances and sizes of the Sun and Moon.

Aristarchus measured the angular diameter of the Moon. The Sun's angular size is difficult to measure because it is so bright, but as we have discussed, during a solar eclipse, the Moon just covers up the Sun, indicating their angular sizes are about the same. We have also discussed how, when the angular size is measured, we can find the distance to the object if its intrinsic size is known. Alternatively, we can find the intrinsic size of an object if its distance is known. These statements can be written mathematically as:

$$S = D\Theta \qquad (1)$$

Where:

- $S =$ The object's intrinsic size
- $D =$ The object's distance
- $\Theta =$ The apparent, or angular size of the object on the sky

Note that this formula has several conditions for its use:

1. The angular size of the object must be small
2. The units for S and D must be the same (e.g., meters and meters)
3. The units for $\Theta$ must be in ***radians***

If you measure the angle $\Theta$ in degrees, you must convert to radians, using

$$1\ \text{radian} = 57.3^{\circ} \qquad (2)$$

Aristarchus knew the angular size, $\Theta$, for the Sun and the Moon. In order to determine their distance (D), he needed to know their diameters (S). Observations of eclipses provided the answer. By observing a lunar eclipse, Aristarchus found that during the eclipse, the Moon moved across the sky an amount equal to 2.5 times the Moon's angular diameter (recall the full Moon is $0.5^{o}$ in angular diameter). Aristarchus also knew that the Sun casts tapering shadows (Figure 1). When there is s total eclipse of the Sun, the Moon just eclipses the whole Sun, and totality is very short – the Moon's shadow only just reaches the Earth (Figure 1). Therefore, over the Earth-Moon distance, the Moon's shadow narrows by the whole Moon diameter. During an eclipse of the Moon, the *Earth's* shadow loses the same amount of width over the same distance. Therefore, Aristarchus concluded :

$$2\,{}^{1}\!/_{2}\ \text{Moon Diameters} = \text{Earth's Diameter - 1 Moon Diameter} \qquad (3)$$

or

$$\text{Earth's Diameter} = 3\,{}^{1}\!/_{2}\ \left(= {}^{7}\!/_{2}\right)\ \text{Moon Diameters} \qquad (4)$$

or

$$\text{Moon's Diameter} = \left({}^{2}\!/_{7}\right)\ \text{Earth's Diameter} \qquad (5)$$

Therefore, the Moon's diameter is 2/7 that of the Earth. In order to actually determine the true diameter of the Moon, one needs to know the diameter of the Earth.

**Question 1:** Who first determined the size (circumference) of the Earth, and briefly describe how he/she did it?

## The Sun

The Sun poses a more difficult problem. Aristarchus waited until the Moon's phase was exactly a half Moon. Sunlight is then falling on the Moon at a right angle to the observer (Figure 2). Aristarchus measured the angle, X, between the Sun and the Moon in the sky (Figure 2). He found the angle to be $87^o$. From tigonometry (Yuk!), the cosine of X is equal to the Earth-Moon distance ($D_{Moon}$) divided by the Earth-Sun distance ($D_{Sun}$).

$$\mathrm{Cos}(X) = \mathrm{D}_{\mathrm{Moon}} / \mathrm{D}_{\mathrm{Sun}} \tag{6}$$

The cosine of $87^o$ is about 1/20, meaning that Aristarchus determined that the Moon was 1/20 of the distance to the Sun. Once the diameter of the Moon is known (Equation 5), one can calculate the Moon's distance (Equation 1), and thus calculate the Sun's distance (Equation 6). Once the Sun's distance is known, one can then calculate it's true size (Equation 1).

Aristarchus' estimates for the Sun were way off. The problem is the difficulty in measuring the angle X. A small error in measuring X gives a large error in the ratio $D_{Moon}/D_{Sun}$.

# PROCEDURE

**Question 2:** Assume the angular size of the Moon is $1/2^o$. How many radians is this?

**Question 3:** Using the modern value for the Earth's diameter (12,756 km), what would Aristarchus have found for the Moon's diameter? The modern value for the Moon's diameter is 3476 km. How accurate is Aristarchus' procedure?

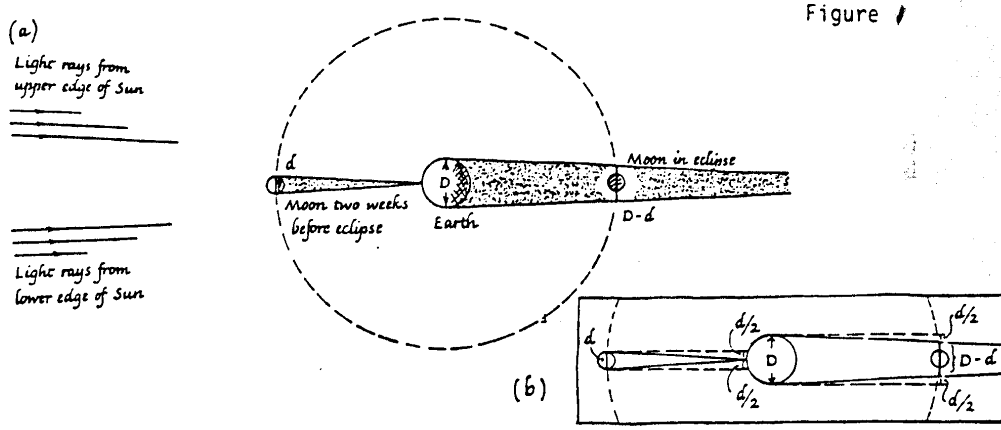
**Question 4:** Using your values for $\Theta$, and Aristarchus' estimate for the diameter of the Moon, what is the distance of the Moon? The modern value for the distance to the Moon is about 384,400 km. How well does your answer agree with the modern value?

**Question 5:** What would Aristarcus have estimated for the distance to the Sun?

**Question 6:** Using a more accurate estimate of the angle X ($X=89.83^{o}$), find the distance to the Sun using Aristarchus' method. The modern value for the distance to the Sun is $1.496 x 10^{8}$ km. How well does your answer agree with the modern value?

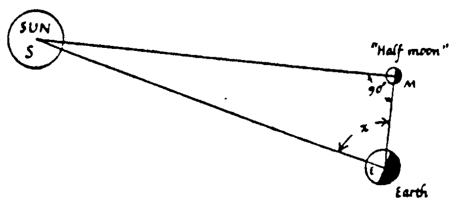
**Question 7:** What is the diameter of the Sun, using your results from Question 6? The modern value for the diameter of the Sun is 1.392x106 km. How well does your answer agree with the modern value?

Figure 1




EARLY GREEK MEASUREMENT OF SIZE OF THE MOON (AND THEREFORE ITS DISTANCE). Observations of eclipses showed that the width of the Earth's shadow at the Moon is 2.5 Moon-diameters. However, the Earth's shadow narrows as its distance from Earth increases because the Sun is not a point-source. Since the Moon's shadow almost dies out in the Moon-Earth distance, the Earth's shadow must narrow by the same amount—one Moon-diameter—in the same distance. Then Earth-diameter must be 3.5 Moon-diameters.

Figure 2




SUN'S DISTANCE
Early Greek estimate of the Sun's distance from the Earth, in terms of the Moon's known distance. Greek astronomers tried to measure the angle x