2 Measurements and Calculations

An enlarged view of a graduated cylinder *(Masterfile)*

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As we pointed out in Chapter 1, making observations is a key part of the scientific process. Sometimes observations are *qualitative* ("the substance is a yellow solid") and sometimes they are *quantitative* ("the substance weighs 4.3 grams"). A quantitative observation is called a **measurement.** Measurements are very important in our daily lives. For example, we pay for gasoline by the gallon, so the gas pump must accurately measure the gas delivered to our fuel tank. The efficiency of the modern automobile engine depends on various measurements, including the amount of oxygen in the exhaust gases, the temperature of the coolant, and the pressure of the lubricating oil. In addition, cars with traction control systems have devices to measure and compare the rates of rotation of all four wheels. As we will see in the "Chemistry in Focus" discussion in this chapter, measuring devices have become very sophisticated in dealing with our fast-moving and complicated society.

As we will discuss in this chapter, a measurement always consists of two parts: a number and a unit. Both parts are necessary to make the measurement meaningful. For example, suppose a friend tells you that she saw a bug 5 long. This statement is meaningless as it stands. Five what? If it's 5 millimeters, the bug is quite small. If it's 5 centimeters, the bug is quite large. If it's 5 meters, run for cover!

The point is that for a measurement to be meaningful, it must consist of both a number and a unit that tells us the scale being used.

In this chapter we will consider the characteristics of measurements and the calculations that involve measurements.

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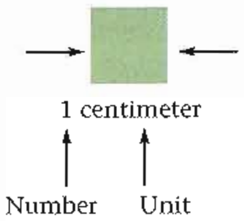
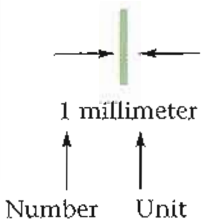
A gas pump measures the amount of gasoline delivered.

2.1 Scientific Notation

OBJECTIVE: To show how very large or very small numbers can be expressed as the product of a number between 1 and 10 and a power of 10.

A measurement must always consist of a number *and* a unit.





The numbers associated with scientific measurements are often very large or very small. For example, the distance from the earth to the sun is approximately 93,000,000 (93 million) miles. Written out, this number is rather bulky. Scientific notation is a method for making very large or very small numbers more compact and easier to write.

To see how this is done, consider the number 125, which can be written as the product

$$125 = 1.25 \times 100$$

Because $100 = 10 \times 10 = 10^2$, we can write

$$125 = 1.25 \times 100 = 1.25 \times 10^2$$

Similarly, the number 1700 can be written

$$1700 = 1.7 \times 1000$$

and because $1000 = 10 \times 10 \times 10 = 10^3$, we can write

$$1700 = 1.7 \times 1000 = 1.7 \times 10^3$$

Ray Simons/Photo Researchers, Inc.

When describing very small distances, such as the diameter of a swine flu virus (shown here magnified 16,537 times), it is convenient to use scientific notation.

MATH SKILL BUILDER
Keep one digit to the left of the decimal point.

Scientific notation simply expresses a number as *a product of a number between 1 and 10 and the appropriate power of 10.* For example, the number 93,000,000 can be expressed as

$$93{,}000{,}000 = 9.3 \times 10{,}000{,}000 = \underbrace{9.3}_{\substack{\text{Number}\\ \text{between}\\ \text{1 and 10}}} \times \underbrace{10^7}_{\substack{\text{Appropriate}\\ \text{power of 10}\\ (10{,}000{,}000\,=\,10^7)}}$$

The easiest way to determine the appropriate power of 10 for scientific notation is to start with the number being represented and count the number of places the decimal point must be moved to obtain a number between 1 and 10. For example, for the number

$$9\;\underbrace{3\;0\;0\;0\;0\;0\;0}_{7\;6\;5\;4\;3\;2\;1}$$

we must move the decimal point seven places to the left to get 9.3 (a number between 1 and 10). To compensate for every move of the decimal point to the left, we must multiply by 10. That is, each time we move the decimal point to the left, we make the number smaller by one power of 10. So for each move of the decimal point to the left, we must multiply by 10 to restore the number to its original magnitude. Thus moving the decimal point seven places to the left means we must multiply 9.3 by 10 seven times, which equals 10^7:

$$93{,}000{,}000 = \underbrace{9.3 \times 10^7}_{\substack{\text{We moved the decimal point seven}\\ \text{places to the left, so we need}\\ 10^7\text{ to compensate.}}}$$

MATH SKILL BUILDER
Moving the decimal point to the left requires a positive exponent.

Remember: whenever the decimal point is moved to the *left,* the exponent of 10 is *positive.*

We can represent numbers smaller than 1 by using the same convention, but in this case the power of 10 is negative. For example, for the number 0.010 we must move the decimal point two places to the right to obtain a number between 1 and 10:

$$0.\underbrace{0\;1}_{1\;2}\;0$$

MATH SKILL BUILDER
Moving the decimal point to the right requires a negative exponent.

This requires an exponent of -2, so $0.010 = 1.0 \times 10^{-2}$. Remember: whenever the decimal point is moved to the *right,* the exponent of 10 is *negative.*

Next consider the number 0.000167. In this case we must move the decimal point four places to the right to obtain 1.67 (a number between 1 and 10):

$$0.\underbrace{0\;0\;0\;1}_{1\;2\;3\;4}.6\;7$$

MATH SKILL BUILDER
Read the Appendix if you need a further discussion of exponents and scientific notation.

Moving the decimal point four places to the right requires an exponent of -4. Therefore,

$$0.000167 = \underbrace{1.67 \times 10^{-4}}_{\substack{\text{We moved the decimal}\\ \text{point four places to the right.}}}$$

We summarize these procedures below.

MATH SKILL BUILDER
$100 = 1.0 \times 10^2$
$0.010 = 1.0 \times 10^{-2}$

MATH SKILL BUILDER
Left Is Positive; remember LIP.

Using Scientific Notation

- Any number can be represented as the product of a number between 1 and 10 and a power of 10 (either positive or negative).
- The power of 10 depends on the number of places the decimal point is moved and in which direction. The *number of places* the decimal point is moved determines the *power of 10*. The *direction* of the move determines whether the power of 10 is *positive* or *negative*. If the decimal point is moved to the left, the power of 10 is positive; if the decimal point is moved to the right, the power of 10 is negative.

EXAMPLE 2.1 Scientific Notation: Powers of 10 (Positive)

MATH SKILL BUILDER
A number that is greater than 1 will always have a positive exponent when written in scientific notation.

Represent the following numbers in scientific notation.

a. 238,000

b. 1,500,000

SOLUTION

a. First we move the decimal point until we have a number between 1 and 10, in this case 2.38.

2 3 8 0 0 0
5 4 3 2 1 — The decimal point was moved five places to the left.

Because we moved the decimal point five places to the left, the power of 10 is positive 5. Thus $238{,}000 = 2.38 \times 10^5$.

b. 1 5 0 0 0 0 0
6 5 4 3 2 1 — The decimal point was moved six places to the left, so the power of 10 is 6.

Thus $1{,}500{,}000 = 1.5 \times 10^6$. ■

EXAMPLE 2.2 Scientific Notation: Powers of 10 (Negative)

MATH SKILL BUILDER
A number that is less than 1 will always have a negative exponent when written in scientific notation.

Represent the following numbers in scientific notation.

a. 0.00043

b. 0.089

SOLUTION

a. First we move the decimal point until we have a number between 1 and 10, in this case 4.3.

0 . 0 0 0 4 3
1 2 3 4 — The decimal point was moved four places to the right.

Because we moved the decimal point four places to the right, the power of 10 is negative 4. Thus $0.00043 = 4.3 \times 10^{-4}$.

b. $0.\underbrace{08}_{1\,2}9$

The power of 10 is negative 2 because the decimal point was moved two places to the right.

Thus $0.089 = 8.9 \times 10^{-2}$.

Self-Check **EXERCISE 2.1** Write the numbers 357 and 0.0055 in scientific notation. If you are having difficulty with scientific notation at this point, reread the Appendix.

See Problems 2.5 through 2.14. ■

2.2 Units

OBJECTIVE: To learn the English, metric, and SI systems of measurement.

The **units** part of a measurement tells us what *scale* or *standard* is being used to represent the results of the measurement. From the earliest days of civilization, trade has required common units. For example, if a farmer from one region wanted to trade some of his grain for the gold of a miner who lived in another region, the two people had to have common standards (units) for measuring the amount of the grain and the weight of the gold.

The need for common units also applies to scientists, who measure quantities such as mass, length, time, and temperature. If every scientist had her or his own personal set of units, complete chaos would result. Unfortunately, although standard systems of units did arise, different systems were adopted in different parts of the world. The two most widely used systems are the **English system** used in the United States and the **metric system** used in most of the rest of the industrialized world.

The metric system has long been preferred for most scientific work. In 1960 an international agreement set up a comprehensive system of units called the **International System** (*le Système Internationale* in French), or **SI.** The SI units are based on the metric system and units derived from the metric system. The most important fundamental SI units are listed in Table 2.1. Later in this chapter we will discuss how to manipulate some of these units.

Because the fundamental units are not always a convenient size, the SI system uses prefixes to change the size of the unit. The most commonly used prefixes are listed in Table 2.2. Although the fundamental unit for length is the meter (m), we can also use the decimeter (dm), which represents one-tenth (0.1) of a meter; the centimeter (cm), which represents one one-hundredth (0.01) of a meter; the millimeter (mm), which represents one one-thousandth (0.001) of a meter; and so on. For example, it's much more convenient to specify the diameter of a certain contact lens as 1.0 cm than as 1.0×10^{-2} m.

Table 2.1 Some Fundamental SI Units

Physical Quantity	Name of Unit	Abbreviation
mass	kilogram	kg
length	meter	m
time	second	s
temperature	kelvin	K

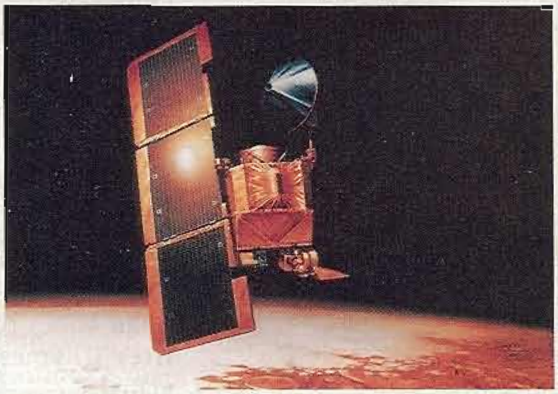
Critical Units!

How important are conversions from one unit to another? If you ask the National Aeronautics and Space Administration (NASA), very important! In 1999 NASA lost a $125 million Mars Climate Orbiter because of a failure to convert from English to metric units.

The problem arose because two teams working on the Mars mission were using different sets of units. NASA's scientists at the Jet Propulsion Laboratory in Pasadena, California, assumed that the thrust data for the rockets on the Orbiter they received from Lockheed Martin Astronautics in Denver, which built the spacecraft, were in metric units. In reality, the units were English. As a result the Orbiter dipped 100 kilometers lower into the Mars atmosphere than planned and the friction from the atmosphere caused the craft to burn up.

NASA's mistake refueled the controversy over whether Congress should require the United States to switch to the metric system. About 95% of the world now uses the metric system, and the United States is slowly switching from English to metric. For example, the automobile industry has adopted metric fasteners and we buy our soda in two-liter bottles.

Units can be very important. In fact, they can mean the difference between life and death on some occasions. In 1983, for example, a Canadian jetliner almost ran out of fuel when someone pumped 22,300 pounds of fuel into the aircraft instead of 22,300 kilograms. Remember to watch your units!



NASA

Artist's conception of the lost Mars Climate Orbiter.

Table 2.2 The Commonly Used Prefixes in the Metric System

Prefix	Symbol	Meaning	Power of 10 for Scientific Notation
mega	M	1,000,000	10^6
kilo	k	1000	10^3
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}

Measurements consist of both a number and a unit, and both are crucial. Just as you would not report a measurement without a numerical value, you would not report a measurement without a unit. You already use units in your daily life, whether you tell somebody, "Let's meet in one hour" (hour is the unit), or you and your friends order two pizzas for dinner (pizza is the unit).

2.3 Measurements of Length, Volume, and Mass

OBJECTIVE: To understand the metric system for measuring length, volume, and mass.

The fundamental SI unit of length is the **meter,** which is a little longer than a yard (1 meter = 39.37 inches). In the metric system fractions of a meter or multiples of a meter can be expressed by powers of 10, as summarized in Table 2.3.

The English and metric systems are compared on the ruler shown in Figure 2.1. Note that

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

Other English–metric equivalences are given in Section 2.6.

Volume is the amount of three-dimensional space occupied by a substance. The fundamental unit of volume in the SI system is based on the volume of a cube that measures 1 meter in each of the three directions. That is, each edge of the cube is 1 meter in length. The volume of this cube is

$$1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = (1 \text{ m})^3 = 1 \text{ m}^3$$

or, in words, one cubic meter.

In Figure 2.2 this cube is divided into 1000 smaller cubes. Each of these small cubes represents a volume of 1 dm^3, which is commonly called the **liter** (rhymes with "meter" and is slightly larger than a quart) and abbreviated L.

The meter was originally defined, in the eighteenth century, as one ten-millionth of the distance from the equator to the North Pole and then, in the late nineteenth century, as the distance between two parallel marks on a special metal bar stored in a vault in Paris. More recently, for accuracy and convenience, a definition expressed in terms of light waves has been adopted.

Table 2.3 The Metric System for Measuring Length

Unit	Symbol	Meter Equivalent
kilometer	km	1000 m or 10^3 m
meter	m	1 m
decimeter	dm	0.1 m or 10^{-1} m
centimeter	cm	0.01 m or 10^{-2} m
millimeter	mm	0.001 m or 10^{-3} m
micrometer	μm	0.000001 m or 10^{-6} m
nanometer	nm	0.000000001 m or 10^{-9} m

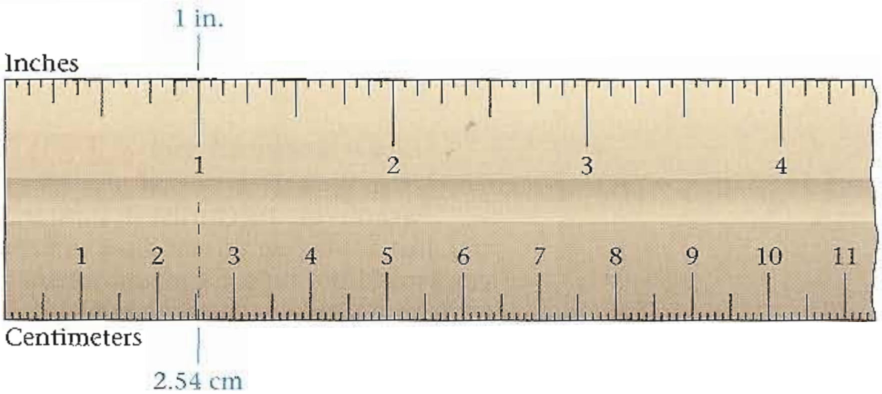


Figure 2.1
Comparison of English and metric units for length on a ruler.

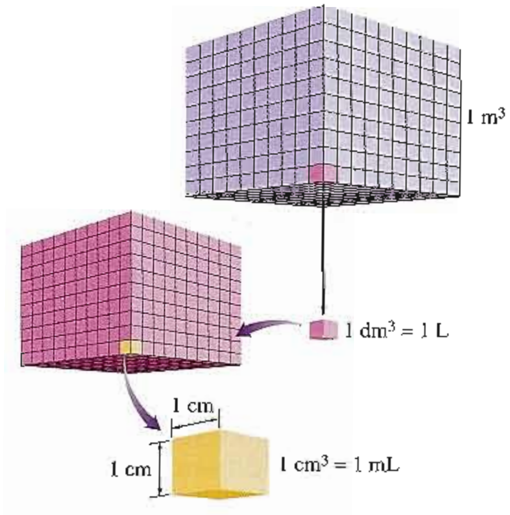


Figure 2.2
The largest drawing represents a cube that has sides 1 m in length and a volume of 1 m^3. The middle-size cube has sides 1 dm in length and a volume of 1 dm^3, or 1 L. The smallest cube has sides 1 cm in length and a volume of 1 cm^3, or 1 mL.

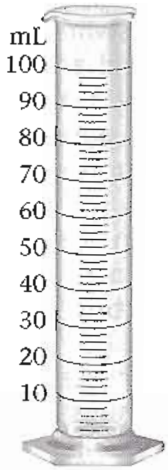


Figure 2.3
A 100-mL graduated cylinder.

Figure 2.4
An electronic analytical balance used in chemistry labs.

The cube with a volume of 1 dm^3 (1 liter) can in turn be broken into 1000 smaller cubes, each representing a volume of 1 cm^3. This means that each liter contains 1000 cm^3. One cubic centimeter is called a **milliliter** (abbreviated mL), a unit of volume used very commonly in chemistry. This relationship is summarized in Table 2.4.

The *graduated cylinder* (see Figure 2.3), commonly used in chemical laboratories for measuring the volumes of liquids, is marked off in convenient units of volume (usually milliliters). The graduated cylinder is filled to the desired volume with the liquid, which then can be poured out.

Another important measurable quantity is **mass,** which can be defined as the quantity of matter present in an object. The fundamental SI unit of mass is the **kilogram.** Because the metric system, which existed before the SI system, used the gram as the fundamental unit, the prefixes for the various mass units are based on the **gram,** as shown in Table 2.5.

In the laboratory we determine the mass of an object by using a balance. A balance compares the mass of the object to a set of standard masses ("weights"). For example, the mass of an object can be determined by using a single-pan balance (Figure 2.4).

To help you get a feeling for the common units of length, volume, and mass, some familiar objects are described in Table 2.6.

Table 2.4 The Relationship of the Liter and Milliliter

Unit	Symbol	Equivalence
liter	L	1 L = 1000 mL
milliliter	mL	$\frac{1}{1000}$ L = 10^{-3} L = 1 mL

Table 2.5 The Most Commonly Used Metric Units for Mass

Unit	Symbol	Gram Equivalent
kilogram	kg	1000 g = 10^3 g = 1 kg
gram	g	1 g
milligram	mg	0.001 g = 10^{-3} g = 1 mg

Measurement: Past, Present, and Future

Measurement lies at the heart of doing science. We obtain the data for formulating laws and testing theories by doing measurements. Measurements also have very practical importance; they tell us if our drinking water is safe, whether we are anemic, and the exact amount of gasoline we put in our cars at the filling station.

Although the fundamental measuring devices we consider in this chapter are still widely used, new measuring techniques are being developed every day to meet the challenges of our increasingly sophisticated world. For example, engines in modern automobiles have oxygen sensors that analyze the oxygen content in the exhaust gases. This information is sent to the computer that controls the engine functions so that instantaneous adjustments can be made in spark timing and air–fuel mixtures to provide efficient power with minimum air pollution.

As another example, consider airline safety: How do we rapidly, conveniently, and accurately determine whether a given piece of baggage contains an explosive device? A thorough hand-search of each piece of luggage is out of the question. Scientists are now developing a screening procedure that bombards the luggage with high-energy particles that cause any substance present to emit radiation characteristic of that substance. This radiation is monitored to identify luggage with unusually large quantities of nitrogen, because most chemical explosives are based on compounds containing nitrogen.


Ben Osborne/Stone/Getty Images

A pollution control officer measuring the oxygen content of river water.

Scientists are also examining the natural world to find supersensitive detectors because many organisms are sensitive to tiny amounts of chemicals in their environments—recall, for example, the sensitive noses of bloodhounds. One of these natural measuring devices uses the sensory hairs from Hawaiian red swimming crabs, which are connected to electrical analyzers and used to detect hormones down to levels of 10^{-8} g/L. Likewise, tissues from pineapple cores can be used to detect tiny amounts of hydrogen peroxide.

These types of advances in measuring devices have led to an unexpected problem: detecting all kinds of substances in our food and drinking water scares us. Although these substances were always there, we didn't worry so much when we couldn't detect them. Now that we know they are present what should we do about them? How can we assess whether these trace substances are harmful or benign? Risk assessment has become much more complicated as our sophistication in taking measurements has increased.

Table 2.6 Some Examples of Commonly Used Units

length	A dime is 1 mm thick.
	A quarter is 2.5 cm in diameter.
	The average height of an adult man is 1.8 m.
mass	A nickel has a mass of about 5 g.
	A 120-lb woman has a mass of about 55 kg.
volume	A 12-oz can of soda has a volume of about 360 mL.
	A half gallon of milk is equal to about 2 L of milk.

2.4 Uncertainty in Measurement

OBJECTIVES: To understand how uncertainty in a measurement arises. • To learn to indicate a measurement's uncertainty by using significant figures.


Andrew Lambert/Leslie Garland Picture Library/Alamy Images

A student performing a titration in the laboratory.

When you measure the amount of something by counting, the measurement is exact. For example, if you asked your friend to buy four apples from the store and she came back with three or five apples, you would be surprised. However, measurements are not always exact. For example, whenever a measurement is made with a device such as a ruler or a graduated cylinder, an estimate is required. We can illustrate this by measuring the pin shown in Figure 2.5a. We can see from the ruler that the pin is a little longer than 2.8 cm and a little shorter than 2.9 cm. Because there are no graduations on the ruler between 2.8 and 2.9, we must estimate the pin's length between 2.8 and 2.9 cm. We do this by *imagining* that the distance between 2.8 and 2.9 is broken into 10 equal divisions (Figure 2.5b) and estimating to which division the end of the pin reaches. The end of the pin appears to come about halfway between 2.8 and 2.9, which corresponds to 5 of our 10 imaginary divisions. So we estimate the pin's length as 2.85 cm. The result of our measurement is that the pin is approximately 2.85 cm in length, but we had to rely on a visual estimate, so it might actually be 2.84 or 2.86 cm.

Because the last number is based on a visual estimate, it may be different when another person makes the same measurement. For example, if five different people measured the pin, the results might be

Person	Result of Measurement
1	2.85 cm
2	2.84 cm
3	2.86 cm
4	2.85 cm
5	2.86 cm

Note that the first two digits in each measurement are the same regardless of who made the measurement; these are called the *certain* numbers of the measurement. However, the third digit is estimated and can vary; it is called an *uncertain* number. When one is making a measurement, the custom is to record all of the certain numbers plus the *first* uncertain number. It would not make any sense to try to measure the pin to the third decimal place (thousandths of a centimeter), because this ruler requires an estimate of even the second decimal place (hundredths of a centimeter).

Every measurement has some degree of uncertainty.

It is very important to realize that *a measurement always has some degree of uncertainty.* The uncertainty of a measurement depends on the

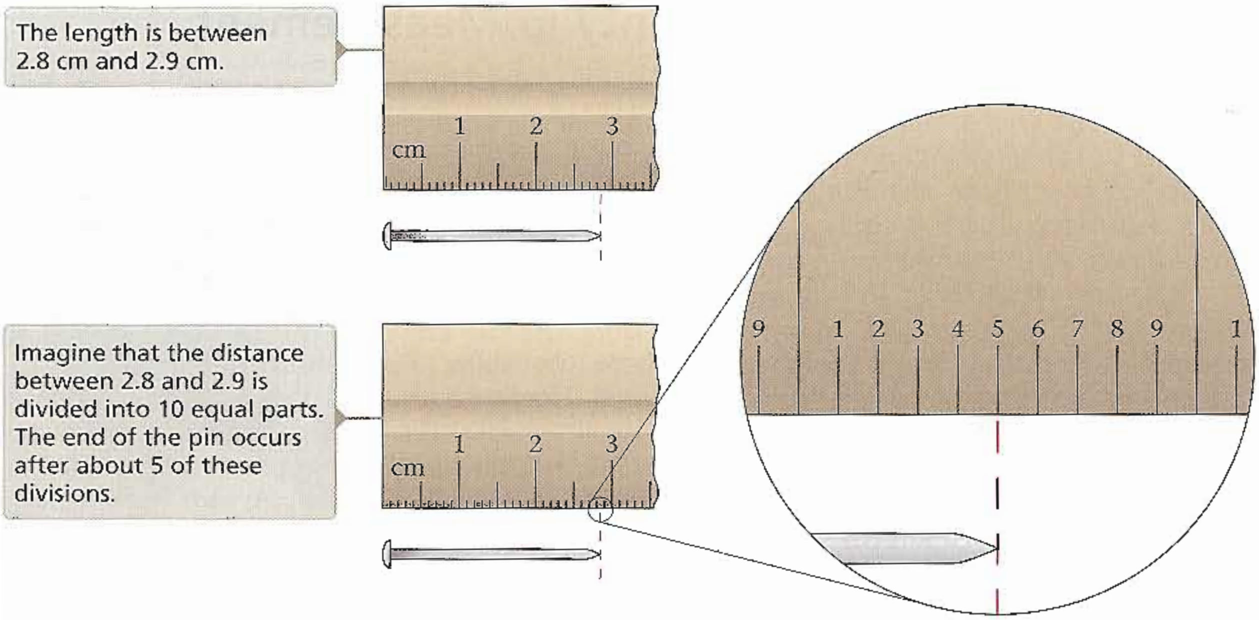


Figure 2.5
Measuring a pin.

measuring device. For example, if the ruler in Figure 2.5 had marks indicating hundredths of a centimeter, the uncertainty in the measurement of the pin would occur in the thousandths place rather than the hundredths place, but some uncertainty would still exist.

The numbers recorded in a measurement (all the certain numbers plus the first uncertain number) are called **significant figures.** The number of significant figures for a given measurement is determined by the inherent uncertainty of the measuring device. For example, the ruler used to measure the pin can give results only to hundredths of a centimeter. Thus, when we record the significant figures for a measurement, we automatically give information about the uncertainty in a measurement. The uncertainty in the last number (the estimated number) is usually assumed to be ± 1 unless otherwise indicated. For example, the measurement 1.86 kilograms can be interpreted as 1.86 ± 0.01 kilograms, where the symbol $\pm$ means plus or minus. That is, it could be 1.86 kg − 0.01 kg = 1.85 kg or 1.86 kg + 0.01 kg = 1.87 kg.

2.5 Significant Figures

OBJECTIVE: To learn to determine the number of significant figures in a calculated result.

We have seen that any measurement involves an estimate and thus is uncertain to some extent. We signify the degree of certainty for a particular measurement by the number of significant figures we record.

Because doing chemistry requires many types of calculations, we must consider what happens when we do arithmetic with numbers that contain uncertainties. It is important that we know the degree of uncertainty in the final result. Although we will not discuss the process here, mathematicians have studied how uncertainty accumulates and have designed a set of rules to determine how many significant figures the result of a calculation should have. You should follow these rules whenever you carry out a calculation. The first thing we need to do is learn how to count the significant figures in a given number. To do this we use the following rules:

Rules for Counting Significant Figures

1. *Nonzero integers.* Nonzero integers *always* count as significant figures. For example, the number 1457 has four nonzero integers, all of which count as significant figures.
2. *Zeros.* There are three classes of zeros:
   a. *Leading zeros* are zeros that *precede* all of the nonzero digits. They *never* count as significant figures. For example, in the number 0.0025, the three zeros simply indicate the position of the decimal point. The number has only two significant figures, the 2 and the 5.
   b. *Captive zeros* are zeros that fall *between* nonzero digits. They *always* count as significant figures. For example, the number 1.008 has four significant figures.
   c. *Trailing zeros* are zeros at the *right end* of the number. They are significant only if the number is written with a decimal point. The number one hundred written as 100 has only one significant figure, but written as 100., it has three significant figures.
3. *Exact numbers.* Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 apples, 8 molecules. Such numbers are called *exact numbers*. They can be assumed to have an unlimited number of significant figures. Exact numbers can also arise from definitions. For example, 1 inch is defined as *exactly* 2.54 centimeters. Thus in the statement 1 in. = 2.54 cm, neither 2.54 nor 1 limits the number of significant figures when it is used in a calculation.

MATH SKILL BUILDER
Leading zeros are never significant figures.

MATH SKILL BUILDER
Captive zeros are always significant figures.

MATH SKILL BUILDER
Trailing zeros are sometimes significant figures.

MATH SKILL BUILDER
Exact numbers never limit the number of significant figures in a calculation.

Rules for counting significant figures also apply to numbers written in scientific notation. For example, the number 100. can also be written as 1.00×10^2, and both versions have three significant figures. Scientific notation offers two major advantages: the number of significant figures can be indicated easily, and fewer zeros are needed to write a very large or a very small number. For example, the number 0.000060 is much more conveniently represented as 6.0×10^{-5}, and the number has two significant figures, written in either form.

MATH SKILL BUILDER
Significant figures are easily indicated by scientific notation.

EXAMPLE 2.3 Counting Significant Figures

Give the number of significant figures for each of the following measurements.

a. A sample of orange juice contains 0.0108 g of vitamin C.

b. A forensic chemist in a crime lab weighs a single hair and records its mass as 0.0050060 g.

c. The distance between two points was found to be 5.030×10^3 ft.

d. In yesterday's bicycle race, 110 riders started but only 60 finished.

SOLUTION

a. The number contains three significant figures. The zeros to the left of the 1 are leading zeros and are not significant, but the remaining zero (a captive zero) is significant.

b. The number contains five significant figures. The leading zeros (to the left of the 5) are not significant. The captive zeros between the 5 and the 6 are significant, and the trailing zero to the right of the 6 is significant because the number contains a decimal point.

c. This number has four significant figures. Both zeros in 5.030 are significant.

d. Both numbers are exact (they were obtained by counting the riders). Thus these numbers have an unlimited number of significant figures.

Self-Check **EXERCISE 2.2** Give the number of significant figures for each of the following measurements.

a. 0.00100 m

b. 2.0800×10^2 L

c. 480 Corvettes

See Problems 2.33 and 2.34.

Rounding Off Numbers

When you perform a calculation on your calculator, the number of digits displayed is usually greater than the number of significant figures that the result should possess. So you must "round off" the number (reduce it to fewer digits). The rules for **rounding off** follow.

These rules reflect the way calculators round off.

Rules for Rounding Off

1. If the digit to be removed
   a. is less than 5, the preceding digit stays the same. For example, 1.33 rounds to 1.3.
   b. is equal to or greater than 5, the preceding digit is increased by 1. For example, 1.36 rounds to 1.4, and 3.15 rounds to 3.2.
2. In a series of calculations, carry the extra digits through to the final result and *then* round off.* This means that you should carry all of the digits that show on your calculator until you arrive at the final number (the answer) and then round off, using the procedures in Rule 1.

We need to make one more point about rounding off to the correct number of significant figures. Suppose the number 4.348 needs to be

*This practice will not be followed in the worked-out examples in this text, because we want to show the correct number of significant figures in each step of the example.

rounded to two significant figures. In doing this, we look *only* at the *first number* to the right of the 3:

$$4.348$$
↑
Look at this number to round off to two significant figures.

MATH SKILL BUILDER
Do not round off sequentially. The number 6.8347 rounded to three significant figures is 6.83, not 6.84.

The number is rounded to 4.3 because 4 is less than 5. It is incorrect to round sequentially. For example, do *not* round the 4 to 5 to give 4.35 and then round the 3 to 4 to give 4.4.

When rounding off, *use only the first number to the right of the last significant figure.*

Determining Significant Figures in Calculations

Next we will learn how to determine the correct number of significant figures in the result of a calculation. To do this we will use the following rules.

Rules for Using Significant Figures in Calculations

1. For *multiplication* or *division*, the number of significant figures in the result is the same as that in the measurement with the *smallest number* of significant figures. We say this measurement is *limiting,* because it limits the number of significant figures in the result. For example, consider this calculation:

$$\underbrace{4.56}_{\text{Three significant figures}} \times \underbrace{1.4}_{\text{Limiting (two significant figures)}} = 6.384 \xrightarrow{\text{Round off}} \underbrace{6.4}_{\text{Two significant figures}}$$

Because 1.4 has only two significant figures, it limits the result to two significant figures. Thus the product is correctly written as 6.4, which has two significant figures. Consider another example. In the division $\frac{8.315}{298}$, how many significant figures should appear in the answer? Because 8.315 has four significant figures, the number 298 (with three significant figures) limits the result. The calculation is correctly represented as

$$\frac{\overbrace{8.315}^{\text{Four significant figures}}}{\underbrace{298}_{\text{Limiting (three significant figures)}}} = \underbrace{0.0279027}_{\text{Result shown on calculator}} \xrightarrow{\text{Round off}} \underbrace{2.79 \times 10^{-2}}_{\text{Three significant figures}}$$

(continued)

MATH SKILL BUILDER
If you need help in using your calculator, see the Appendix.

2. For *addition* or *subtraction*, the limiting term is the one with the smallest number of decimal places. For example, consider the following sum:

$$\begin{array}{r} 12.11 \\ 18.0 \\ \underline{1.013} \\ 31.123 \end{array} \quad \begin{array}{l} \\ \text{Limiting term (has one decimal place)} \\ \\ \xrightarrow{\text{Round off}} 31.1 \end{array}$$

One decimal place

Why is the answer limited by the term with the smallest number of decimal places? Recall that the last digit reported in a measurement is actually an uncertain number. Although 18, 18.0, and 18.00 are treated as the same quantities by your calculator, they are different to a scientist. The problem above can be thought of as follows:

$$\begin{array}{r} 12.11?\ \text{mL} \\ 18.0??\ \text{mL} \\ \underline{1.013\ \text{mL}} \\ 31.1??\ \text{mL} \end{array}$$

Because the term 18.0 is reported only to the tenths place, our answer must be reported this way as well.

The correct result is 31.1 (it is limited to one decimal place because 18.0 has only one decimal place). Consider another example:

$$\begin{array}{r} 0.6875 \\ \underline{-0.1} \\ 0.5875 \end{array} \quad \begin{array}{l} \\ \text{Limiting term (one decimal place)} \\ \\ \xrightarrow{\text{Round off}} 0.6 \end{array}$$

Note that *for multiplication and division, significant figures are counted. For addition and subtraction, the decimal places are counted.*

Now we will put together the things you have learned about significant figures by considering some mathematical operations in the following examples.

EXAMPLE 2.4 Counting Significant Figures in Calculations

Without performing the calculations, tell how many significant figures each answer should contain.

a. $\begin{array}{r} 5.19 \\ 1.9 \\ \underline{+0.842} \end{array}$

b. $1081 - 7.25$

c. 2.3×3.14

d. the total cost of 3 boxes of candy at \$2.50 a box

SOLUTION

a. The answer will have one digit after the decimal place. The limiting number is 1.9, which has one decimal place, so the answer has two significant figures.

b. The answer will have no digits after the decimal point. The number 1081 has no digits to the right of the decimal point and limits the result, so the answer has four significant figures.

c. The answer will have two significant figures because the number 2.3 has only two significant figures (3.14 has three).

d. The answer will have three significant figures. The limiting factor is 2.50 because 3 (boxes of candy) is an exact number. ■

EXAMPLE 2.5 Calculations Using Significant Figures

Carry out the following mathematical operations and give each result to the correct number of significant figures.

a. 5.18×0.0208

b. $(3.60 \times 10^{-3}) \times (8.123) \div 4.3$

c. $21 + 13.8 + 130.36$

d. $116.8 - 0.33$

e. $(1.33 \times 2.8) + 8.41$

SOLUTION

Limiting terms — Round to this digit.

a. $5.18 \times 0.0208 = 0.107744 \Rightarrow 0.108$

The answer should contain three significant figures because each number being multiplied has three significant figures (Rule 1). The 7 is rounded to 8 because the following digit is greater than 5.

Round to this digit.

b. $$\frac{(3.60 \times 10^{-3})(8.123)}{4.3} = 6.8006 \times 10^{-3} \Rightarrow 6.8 \times 10^{-3}$$

Limiting term

Because 4.3 has the least number of significant figures (two), the result should have two significant figures (Rule 1).

MATH SKILL BUILDER
When we multiply and divide in a problem, perform all calculations before rounding the answer to the correct number of significant figures.

c.
$$\begin{array}{r} 21 \\ 13.8 \\ +130.36 \\ \hline 165.16 \end{array} \Rightarrow 165$$

In this case 21 is limiting (there are no digits after the decimal point). Thus the answer must have no digits after the decimal point, in accordance with the rule for addition (Rule 2).

d.
$$\begin{array}{r} 116.8 \\ -\ \ 0.33 \\ \hline 116.47 \end{array} \Rightarrow 116.5$$

Because 116.8 has only one decimal place, the answer must have only one decimal place (Rule 2). The 4 is rounded up to 5 because the digit to the right (7) is greater than 5.

e. $1.33 \times 2.8 = 3.724 \Rightarrow 3.7$

$$\begin{array}{r} 3.7 \leftarrow \text{Limiting term} \\ +\ 8.41 \\ \hline 12.11 \end{array} \Rightarrow 12.1$$

Note that in this case we multiplied and then rounded the result to the correct number of significant figures before we performed the addition so that we would know the correct number of decimal places.

MATH SKILL BUILDER
When we multiply (or divide) and then add (or subtract) in a problem, round the first answer from the first operation (in this case, multiplication) before performing the next operation (in this case, addition). We need to know the correct number of decimal places.

Self-Check EXERCISE 2.3 Give the answer for each calculation to the correct number of significant figures.

a. 12.6×0.53

b. $(12.6 \times 0.53) - 4.59$

c. $(25.36 - 4.15) \div 2.317$

See Problems 2.47 through 2.52.

2.6 Problem Solving and Dimensional Analysis

OBJECTIVE: To learn how dimensional analysis can be used to solve various types of problems.

Suppose that the boss at the store where you work on weekends asks you to pick up 2 dozen doughnuts on the way to work. However, you find that the doughnut shop sells by the doughnut. How many doughnuts do you need?

This "problem" is an example of something you encounter all the time: converting from one unit of measurement to another. Examples of this occur in cooking (The recipe calls for 3 cups of cream, which is sold in pints. How many pints do I buy?); traveling (The purse costs 250 pesos. How much is that in dollars?); sports (A recent Tour de France bicycle race was 3215 kilometers long. How many miles is that?); and many other areas.

How do we convert from one unit of measurement to another? Let's explore this process by using the doughnut problem.

$$2 \text{ dozen doughnuts} = ? \text{ individual doughnuts}$$

where ? represents a number you don't know yet. The essential information you must have is the definition of a dozen:

$$1 \text{ dozen} = 12$$

You can use this information to make the needed conversion as follows:

$$2 \text{ dozen doughnuts} \times \frac{12}{1 \text{ dozen}} = 24 \text{ doughnuts}$$

MATH SKILL BUILDER
Since 1 dozen = 12, when we multiply by $\frac{12}{1 \text{ dozen}}$, we are multiplying by 1. The unit "dozen" cancels.

You need to buy 24 doughnuts.

Note two important things about this process.

1. The factor $\frac{12}{1 \text{ dozen}}$ is a conversion factor based on the definition of the term *dozen*. This conversion factor is a ratio of the two parts of the definition of a dozen given above.
2. The unit "dozen" itself cancels.

Now let's generalize a bit. To change from one unit to another we will use a conversion factor.

$$\text{Unit}_1 \times \text{conversion factor} = \text{Unit}_2$$

The **conversion factor** is a ratio of the two parts of the statement that relates the two units. We will see this in more detail on the following pages.

Earlier in this chapter we considered a pin that measured 2.85 cm in length. What is the length of the pin in inches? We can represent this problem as

$$2.85 \text{ cm} \rightarrow ? \text{ in.}$$

The question mark stands for the number we want to find. To solve this problem, we must know the relationship between inches and centimeters. In Table 2.7, which gives several equivalents between the English and metric systems, we find the relationship

$$2.54 \text{ cm} = 1 \text{ in.}$$

Table 2.7 English–Metric and English–English Equivalents

Length	1 m = 1.094 yd
	2.54 cm = 1 in.
	1 mi = 5280. ft
	1 mi = 1760. yd
Mass	1 kg = 2.205 lb
	453.6 g = 1 lb
Volume	1 L = 1.06 qt
	1 ft^3 = 28.32 L

This is called an **equivalence statement.** In other words, 2.54 cm and 1 in. stand for *exactly the same distance.* (See Figure 2.1.) The respective numbers are different because they refer to different *scales* (*units*) of distance.

The equivalence statement 2.54 cm = 1 in. can lead to either of two conversion factors:

$$\frac{2.54\text{ cm}}{1\text{ in.}} \quad \text{or} \quad \frac{1\text{ in.}}{2.54\text{ cm}}$$

Note that these *conversion factors* are *ratios of the two parts of the equivalence statement* that relates the two units. Which of the two possible conversion factors do we need? Recall our problem:

$$2.85\text{ cm} = ?\text{ in.}$$

That is, we want to convert from units of centimeters to inches:

$$2.85\text{ cm} \times \text{conversion factor} = ?\text{ in.}$$

We choose a conversion factor that cancels the units we want to discard and leaves the units we want in the result. Thus we do the conversion as follows:

MATH SKILL BUILDER
Units cancel just as numbers do.

$$2.85\text{ cm} \times \frac{1\text{ in.}}{2.54\text{ cm}} = \frac{2.85\text{ in.}}{2.54} = 1.12\text{ in.}$$

Note two important facts about this conversion:

1. The centimeter units cancel to give inches for the result. This is exactly what we had wanted to accomplish. Using the other conversion factor $\left(2.85\text{ cm} \times \frac{2.54\text{ cm}}{1\text{ in.}}\right)$ would not work because the units would not cancel to give inches in the result.

MATH SKILL BUILDER
When you finish a calculation, always check to make sure that the answer makes sense.

2. As the units changed from centimeters to inches, the number changed from 2.85 to 1.12. Thus 2.85 cm has exactly the same value (is the same length) as 1.12 in. Notice that in this conversion, the number decreased from 2.85 to 1.12. This makes sense because the inch is a larger unit of length than the centimeter is. That is, it takes fewer inches to make the same length in centimeters.

MATH SKILL BUILDER
When exact numbers are used in a calculation, they never limit the number of significant digits.

The result in the foregoing conversion has three significant figures as required. Caution: Noting that the term 1 appears in the conversion, you might think that because this number appears to have only one significant figure, the result should have only one significant figure. That is, the answer should be given as 1 in. rather than 1.12 in. However, in the equivalence statement 1 in. = 2.54 cm, the 1 is an exact number (by definition). In other words, exactly 1 in. equals 2.54 cm. Therefore, the 1 does not limit the number of significant digits in the result.

We have seen how to convert from centimeters to inches. What about the reverse conversion? For example, if a pencil is 7.00 in. long, what is its length in centimeters? In this case, the conversion we want to make is

$$7.00\text{ in.} \rightarrow ?\text{ cm}$$

What conversion factor do we need to make this conversion?

Remember that two conversion factors can be derived from each equivalence statement. In this case, the equivalence statement 2.54 cm = 1 in. gives

$$\frac{2.54\text{ cm}}{1\text{ in.}} \quad \text{or} \quad \frac{1\text{ in.}}{2.54\text{ cm}}$$

Again, we choose which factor to use by looking at the *direction* of the required change. For us to change from inches to centimeters, the inches must cancel. Thus the factor

Consider the direction of the required change in order to select the correct conversion factor.

$$\frac{2.54 \text{ cm}}{1 \text{ in.}}$$

is used, and the conversion is done as follows:

$$7.00 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = (7.00)(2.54) \text{ cm} = 17.8 \text{ cm}$$

Here the inch units cancel, leaving centimeters as required.

Note that in this conversion, the number increased (from 7.00 to 17.8). This makes sense because the centimeter is a smaller unit of length than the inch. That is, it takes more centimeters to make the same length in inches. *Always take a moment to think about whether your answer makes sense.* This will help you avoid errors.

Changing from one unit to another via conversion factors (based on the equivalence statements between the units) is often called **dimensional analysis.** We will use this method throughout our study of chemistry.

We can now state some general steps for doing conversions by dimensional analysis.

Converting from One Unit to Another

Step 1 To convert from one unit to another, use the equivalence statement that relates the two units. The conversion factor needed is a ratio of the two parts of the equivalence statement.

Step 2 Choose the appropriate conversion factor by looking at the direction of the required change (make sure the unwanted units cancel).

Step 3 Multiply the quantity to be converted by the conversion factor to give the quantity with the desired units.

Step 4 Check that you have the correct number of significant figures.

Step 5 Ask whether your answer makes sense.

We will now illustrate this procedure in Example 2.6.

EXAMPLE 2.6 Conversion Factors: One-Step Problems

An Italian bicycle has its frame size given as 62 cm. What is the frame size in inches?

SOLUTION

We can represent the problem as

$$62 \text{ cm} = ? \text{ in.}$$

In this problem we want to convert from centimeters to inches.

$$62 \text{ cm} \times \text{conversion factor} = ? \text{ in.}$$

Step 1 To convert from centimeters to inches, we need the equivalence statement 1 in. = 2.54 cm. This leads to two conversion factors:

$$\frac{1 \text{ in.}}{2.54 \text{ cm}} \quad \text{and} \quad \frac{2.54 \text{ cm}}{1 \text{ in.}}$$

Step 2 In this case, the direction we want is

Centimeters → inches

so we need the conversion factor $\frac{1 \text{ in.}}{2.54 \text{ cm}}$. We know this is the one we want because using it will make the units of centimeters cancel, leaving units of inches.

Step 3 The conversion is carried out as follows:

$$62 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} = 24 \text{ in.}$$

Step 4 The result is limited to two significant figures by the number 62. The centimeters cancel, leaving inches as required.

Step 5 Note that the number decreased in this conversion. This makes sense; the inch is a larger unit of length than the centimeter.

Self-Check **EXERCISE 2.4** Wine is often bottled in 0.750-L containers. Using the appropriate equivalence statement from Table 2.7, calculate the volume of such a wine bottle in quarts.

See Problems 2.59 and 2.60.

Next we will consider a conversion that requires several steps.

EXAMPLE 2.7

Conversion Factors: Multiple-Step Problems

The length of the marathon race is approximately 26.2 mi. What is this distance in kilometers?

SOLUTION

The problem before us can be represented as follows:

$$26.2 \text{ mi} = ? \text{ km}$$

We could accomplish this conversion in several different ways, but because Table 2.7 gives the equivalence statements 1 mi = 1760 yd and 1 m = 1.094 yd, we will proceed as follows:

Miles → yards → meters → kilometers

This process will be carried out one conversion at a time to make sure everything is clear.

MILES → YARDS: We convert from miles to yards using the conversion factor $\frac{1760 \text{ yd}}{1 \text{ mi}}$.

$$26.2 \cancel{\text{mi}} \times \frac{1760 \text{ yd}}{1 \cancel{\text{mi}}} = 46{,}112 \text{ yd}$$

Result shown on calculator

$$46{,}112 \text{ yd} \xrightarrow{\text{Round off}} 46{,}100 \text{ yd} = 4.61 \times 10^4 \text{ yd}$$

YARDS → METERS: The conversion factor used to convert yards to meters is $\frac{1\text{ m}}{1.094\text{ yd}}$.

$$4.61 \times 10^4\ \cancel{\text{yd}} \times \frac{1\text{ m}}{1.094\ \cancel{\text{yd}}} = 4.213894 \times 10^4\text{ m}$$

Result shown on calculator

$$4.213894 \times 10^4\text{ m} \xrightarrow{\text{Round off}} 4.21 \times 10^4\text{ m}$$

METERS → KILOMETERS: Because 1000 m = 1 km, or 10^3 m = 1 km, we convert from meters to kilometers as follows:

$$4.21 \times 10^4\ \cancel{\text{m}} \times \frac{1\text{ km}}{10^3\ \cancel{\text{m}}} = 4.21 \times 10^1\text{ km}$$
$$= 42.1\text{ km}$$

Thus the marathon (26.2 mi) is 42.1 km.

Once you feel comfortable with the conversion process, you can combine the steps. For the above conversion, the combined expression is

miles → yards → meters → kilometers

$$26.2\ \cancel{\text{mi}} \times \frac{1760\ \cancel{\text{yd}}}{1\ \cancel{\text{mi}}} \times \frac{1\ \cancel{\text{m}}}{1.094\ \cancel{\text{yd}}} \times \frac{1\text{ km}}{10^3\ \cancel{\text{m}}} = 42.1\text{ km}$$

Note that the units cancel to give the required kilometers and that the result has three significant figures.

MATH SKILL BUILDER
Remember that we are rounding off at the end of each step to show the correct number of significant figures. However, in doing a multistep calculation, *you* should retain the extra numbers that show on your calculator and round off only at the end of the calculation.

Self-Check EXERCISE 2.5 Racing cars at the Indianapolis Motor Speedway now routinely travel around the track at an average speed of 225 mi/h. What is this speed in kilometers per hour?

See Problems 2.65 and 2.66.

Units provide a very valuable check on the validity of your solution. Always use them.

Recap: Whenever you work problems, remember the following points:

1. Always include the units (a measurement always has two parts: a number *and* a unit).
2. Cancel units as you carry out the calculations.
3. Check that your final answer has the correct units. If it doesn't, you have done something wrong.
4. Check that your final answer has the correct number of significant figures.
5. Think about whether your answer makes sense.

2.7 Temperature Conversions: An Approach to Problem Solving

OBJECTIVES: To learn the three temperature scales. • To learn to convert from one scale to another. • To continue to develop problem-solving skills.

When the doctor tells you your temperature is 102 degrees and the weatherperson on TV says it will be 75 degrees tomorrow, they are using the **Fahrenheit scale.** Water boils at 212 °F and freezes at 32 °F, and normal body temperature is 98.6 °F (where °F signifies "Fahrenheit degrees"). This temperature scale is widely used in the United States and Great Britain, and it is the scale employed in most of the engineering sciences. Another temperature scale, used in Canada and Europe and in the physical and life sciences in most countries, is the **Celsius scale.** In keeping with the metric system, which is based on powers of 10, the freezing and boiling points of water on the Celsius scale are assigned as 0 °C and 100 °C, respectively. On both the Fahrenheit and the Celsius scales, the unit of temperature is called a degree, and the symbol for it is followed by the capital letter representing the scale on which the units are measured: °C or °F.

Although 373 K is often stated as 373 degrees Kelvin, it is more correct to say 373 kelvins.

Still another temperature scale used in the sciences is the **absolute** or **Kelvin scale.** On this scale water freezes at 273 K and boils at 373 K. On the Kelvin scale, the unit of temperature is called a kelvin and is symbolized by K. Thus, on the three scales, the boiling point of water is stated as 212 Fahrenheit degrees (212 °F), 100 Celsius degrees (100 °C), and 373 kelvins (373 K).

The three temperature scales are compared in Figures 2.6 and 2.7. There are several important facts you should note.

1. The size of each temperature unit (each degree) is the same for the Celsius and Kelvin scales. This follows from the fact that the *difference* between the boiling and freezing points of water is 100 units on both of these scales.
2. The Fahrenheit degree is smaller than the Celsius and Kelvin units. Note that on the Fahrenheit scale there are 180 Fahrenheit degrees between the boiling and freezing points of water, as compared with 100 units on the other two scales.
3. The zero points are different on all three scales.

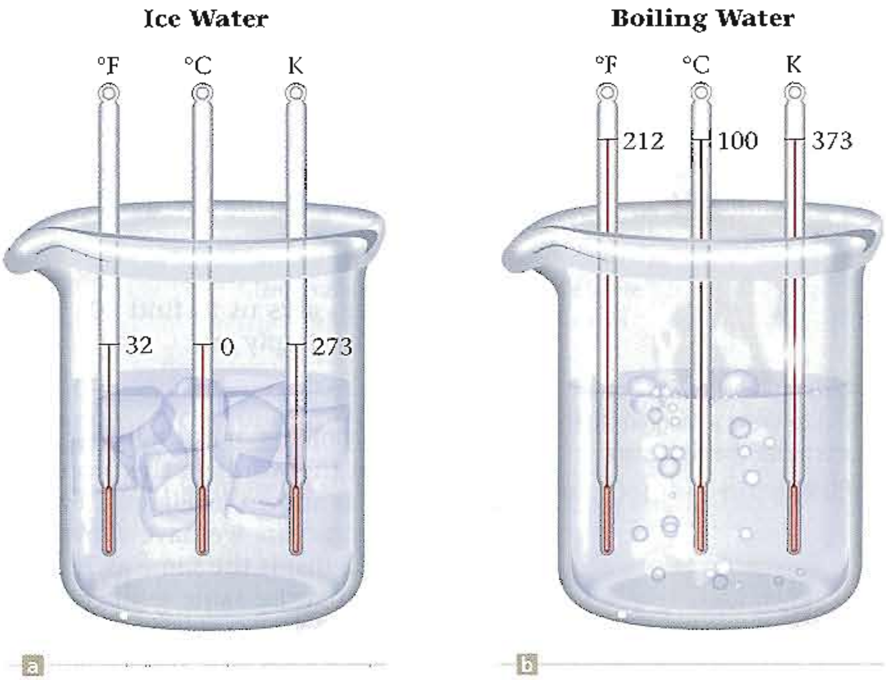


Figure 2.6
Thermometers based on the three temperature scales in (a) ice water and (b) boiling water.

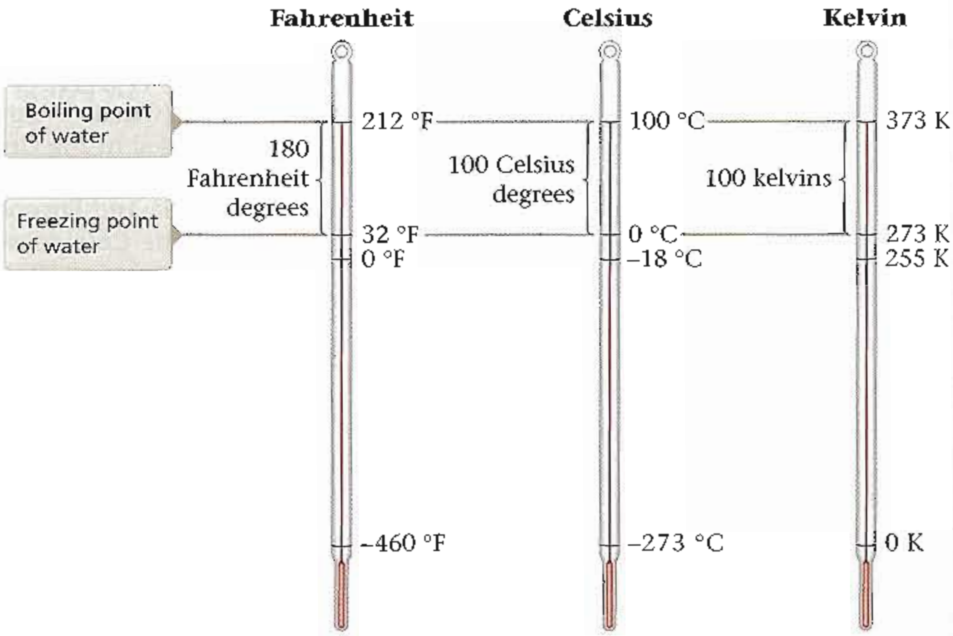


Figure 2.7
The three major temperature scales.

In your study of chemistry, you will sometimes need to convert from one temperature scale to another. We will consider in some detail how this is done. In addition to learning how to change temperature scales, you should also use this section as an opportunity to further develop your skills in problem solving.

Converting Between the Kelvin and Celsius Scales

It is relatively simple to convert between the Celsius and Kelvin scales because the temperature unit is the same size; only the zero points are different. Because 0 °C corresponds to 273 K, converting from Celsius to Kelvin requires that we add 273 to the Celsius temperature. We will illustrate this procedure in Example 2.8.

EXAMPLE 2.8 Temperature Conversion: Celsius to Kelvin

Boiling points will be discussed further in Chapter 14.

The boiling point of water at the top of Mt. Everest is 70. °C. Convert this temperature to the Kelvin scale. (The decimal point after the temperature reading indicates that the trailing zero is significant.)

SOLUTION

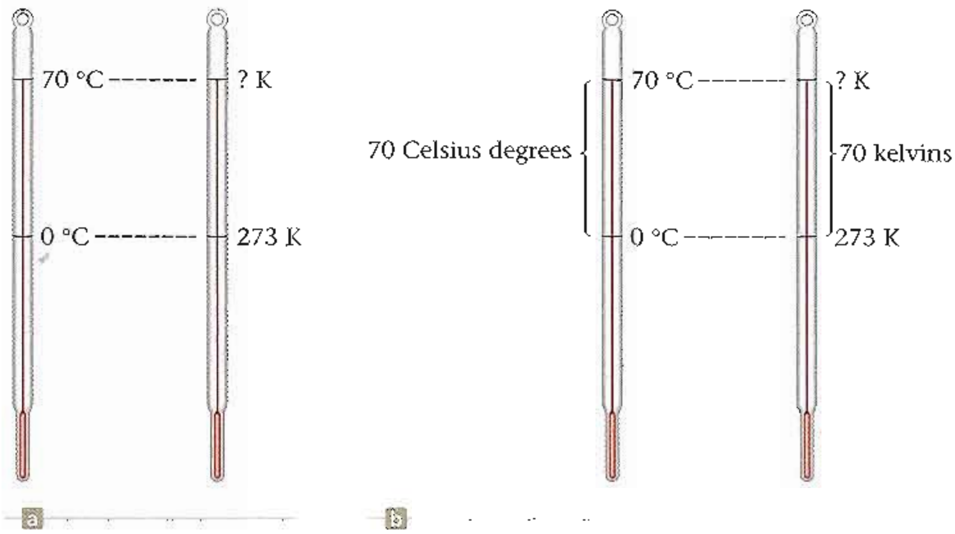
This problem asks us to find 70. °C in units of kelvins. We can represent this problem simply as

$$70.\ °C = ?\ K$$

In solving problems, it is often helpful to draw a diagram that depicts what the words are telling you.

In doing problems, it is often helpful to draw a diagram in which we try to represent the words in the problem with a picture. This problem can be diagramed as shown in Figure 2.8a.

In this picture we have shown what we want to find: "What temperature (in kelvins) is the same as 70. °C?" We also know from Figure 2.7 that 0 °C represents the same temperature as 273 K. How many degrees above 0 °C is 70. °C? The answer, of course, is 70. Thus we must add 70. to 0 °C to reach 70. °C. Because degrees are the *same size* on both the Celsius scale



We know 0 °C = 273 K. We want to know 70. °C = ? K.

There are 70 degrees on the Celsius scale between 0 °C and 70. °C. Because units on these scales are the same size, there are also 70 kelvins in this same distance on the Kelvin scale.

Figure 2.8 Converting 70. °C to units measured on the Kelvin scale.

and the Kelvin scale (see Figure 2.8b), we must also add 70. to 273 K (same temperature as 0 °C) to reach ? K. That is,

$$? \text{ K} = 273 + 70. = 343 \text{ K}$$

Thus 70. °C corresponds to 343 K.

Note that to convert from the Celsius to the Kelvin scale, we simply add the temperature in °C to 273. That is,

$$\underbrace{T_{°C}}_{\text{Temperature in Celsius degrees}} + 273 = \underbrace{T_{K}}_{\text{Temperature in kelvins}}$$

Using this formula to solve the present problem gives

$$70. + 273 = 343$$

(with units of kelvins, K), which is the correct answer. ■

We can summarize what we learned in Example 2.8 as follows: to convert from the Celsius to the Kelvin scale, we can use the formula

$$\underbrace{T_{°C}}_{\text{Temperature in Celsius degrees}} + 273 = \underbrace{T_{K}}_{\text{Temperature in kelvins}}$$

EXAMPLE 2.9 Temperature Conversion: Kelvin to Celsius

Liquid nitrogen boils at 77 K. What is the boiling point of nitrogen on the Celsius scale?

SOLUTION

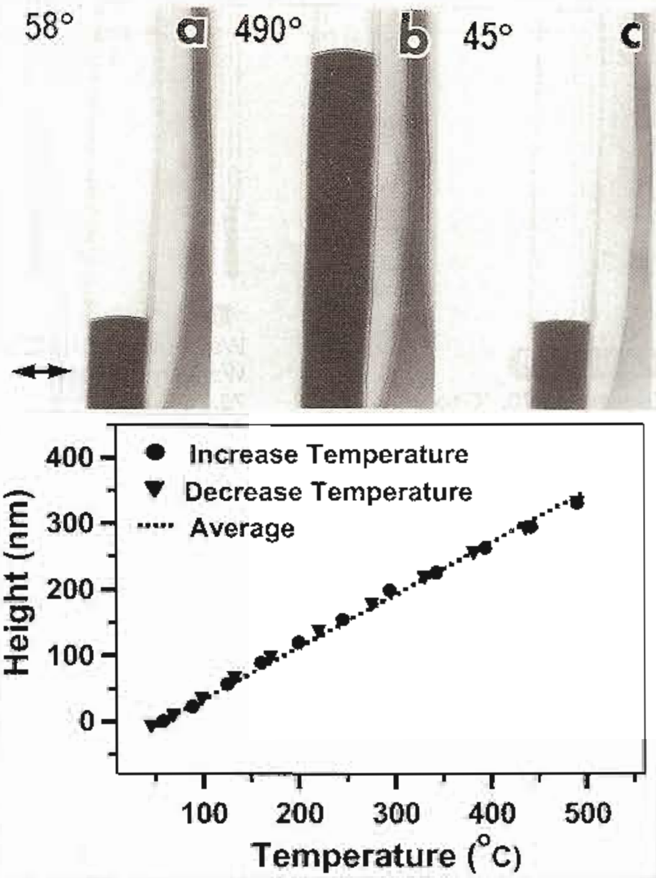
The problem to be solved here is 77 K = ? °C. Let's explore this question by examining the picture on the following page representing the two

Tiny Thermometers

Can you imagine a thermometer that has a diameter equal to one one-hundredth of a human hair? Such a device has actually been produced by scientists Yihica Gao and Yoshio Bando of the National Institute for Materials Science in Tsukuba, Japan. The thermometer they constructed is so tiny that it must be read using a powerful electron microscope.

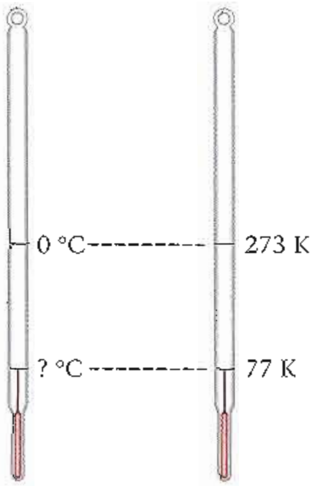
It turns out that the tiny thermometers were produced by accident. The Japanese scientists were actually trying to make tiny (nanoscale) gallium nitride wires. However, when they examined the results of their experiment, they discovered tiny tubes of carbon atoms that were filled with elemental gallium. Because gallium is a liquid over an unusually large temperature range, it makes a perfect working liquid for a thermometer. Just as in mercury thermometers, which have mostly been phased out because of the toxicity of mercury, the gallium expands as the temperature increases. Therefore, gallium moves up the tube as the temperature increases.

These minuscule thermometers are not useful in the normal macroscopic world—they can't even be seen with the naked eye. However, they should be valuable for monitoring temperatures from 50 °C to 500 °C in materials in the nanoscale world.



Liquid gallium expands within a carbon nanotube as the temperature increases (left to right).

Dr. Yoshio Bando/National Institute for Materials Sciences



temperature scales. One key point is to recognize that 0 °C = 273 K. Also note that the difference between 273 K and 77 K is 196 kelvins (273 − 77 = 196). That is, 77 K is 196 kelvins below 273 K. The degree size is the same on these two temperature scales, so 77 K must correspond to 196 Celsius degrees below zero or −196 °C. Thus 77 K = ? °C = −196 °C.

We can also solve this problem by using the formula

$$T_{°C} + 273 = T_K$$

However, in this case we want to solve for the Celsius temperature, $T_{°C}$. That is, we want to isolate $T_{°C}$ on one side of the equals sign. To do this we use an important general principle: doing *the same thing on both sides of the equals sign* preserves the equality. In other words, it's always okay to perform the same operation on both sides of the equals sign.

To isolate $T_{°C}$ we need to subtract 273 from both sides:

$$T_{°C} + \underbrace{273 - 273}_{\text{Sum is zero}} = T_K - 273$$

to give

$$T_{°C} = T_K - 273$$

Using this equation to solve the problem, we have

$$T_{°C} = T_K - 273 = 77 - 273 = -196$$

So, as before, we have shown that

$$77 \text{ K} = -196 \text{ °C}$$

Self-Check **EXERCISE 2.6** Which temperature is colder, 172 K or −75 °C?

See Problems 2.73 and 2.74.

In summary, because the Kelvin and Celsius scales have the same size unit, to switch from one scale to the other we must simply account for the different zero points. We must add 273 to the Celsius temperature to obtain the temperature on the Kelvin scale:

$$T_K = T_{°C} + 273$$

To convert from the Kelvin scale to the Celsius scale, we must subtract 273 from the Kelvin temperature:

$$T_{°C} = T_K - 273$$

Converting Between the Fahrenheit and Celsius Scales

The conversion between the Fahrenheit and Celsius temperature scales requires two adjustments:

1. For the different size units
2. For the different zero points

To see how to adjust for the different unit sizes, consider the diagram in Figure 2.9. Note that because 212 °F = 100 °C and 32 °F = 0 °C,

$$212 - 32 = 180 \text{ Fahrenheit degrees} = 100 - 0 = 100 \text{ Celsius degrees}$$

Thus

$$180. \text{ Fahrenheit degrees} = 100. \text{ Celsius degrees}$$

MATH SKILL BUILDER
Remember, it's okay to do the same thing to both sides of the equation.

Dividing both sides of this equation by 100. gives

$$\frac{180.}{100.} \text{ Fahrenheit degrees} = \frac{\cancel{100.}}{\cancel{100.}} \text{ Celsius degrees}$$

or

$$1.80 \text{ Fahrenheit degrees} = 1.00 \text{ Celsius degree}$$

The factor 1.80 is used to convert from one degree size to the other.

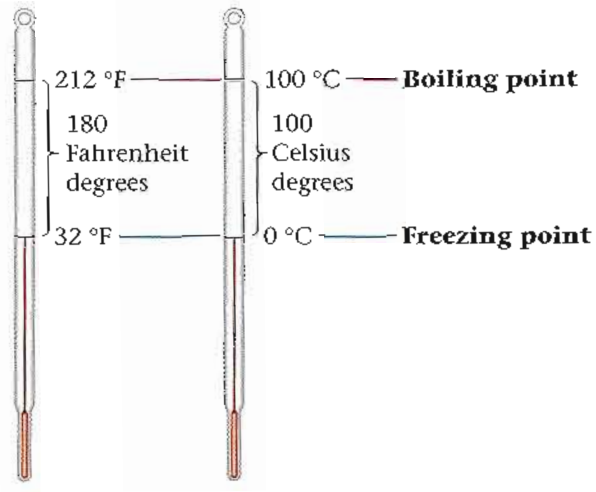


Figure 2.9
Comparison of the Celsius and Fahrenheit scales.

Next we have to account for the fact that 0 °C is *not* the same as 0 °F. In fact, 32 °F = 0 °C. Although we will not show how to derive it, the equation to convert a temperature in Celsius degrees to the Fahrenheit scale is

$$\underbrace{T_{°F}}_{\text{Temperature in °F}} = 1.80(\underbrace{T_{°C}}_{\text{Temperature in °C}}) + 32$$

In this equation the term $1.80(T_{°C})$ adjusts for the difference in degree size between the two scales. The 32 in the equation accounts for the different zero points. We will now show how to use this equation.

EXAMPLE 2.10

Temperature Conversion: Celsius to Fahrenheit

On a summer day the temperature in the laboratory, as measured on a lab thermometer, is 28 °C. Express this temperature on the Fahrenheit scale.

SOLUTION

This problem can be represented as 28 °C = ? °F. We will solve it using the formula

$$T_{°F} = 1.80\,(T_{°C}) + 32$$

In this case,

$$T_{°F} = ?\ °F = 1.80(\overset{T_{°C}}{\overset{\downarrow}{28}}) + 32 = \underbrace{50.4}_{\text{Rounds off to 50}} + 32$$

$$= 50. + 32 = 82$$

Thus 28 °C = 82 °F. ■

Note that 28 °C is approximately equal to 82 °F. Because the numbers are just reversed, this is an easy reference point to remember for the two scales.

EXAMPLE 2.11 Temperature Conversion: Celsius to Fahrenheit

Express the temperature −40. °C on the Fahrenheit scale.

SOLUTION

We can express this problem as −40. °C = ? °F. To solve it we will use the formula

$$T_{°F} = 1.80\,(T_{°C}) + 32$$

In this case,

$$\begin{aligned} T_{°F} = ?\,°F &= 1.80(\overset{T_{°C}}{\overset{\downarrow}{-40.}}) + 32 \\ &= -72 + 32 = -40 \end{aligned}$$

So −40 °C = −40 °F. This is a very interesting result and is another useful reference point.

Self-Check **EXERCISE 2.7** Hot tubs are often maintained at 41 °C. What is this temperature in Fahrenheit degrees?

See Problems 2.75 through 2.78. ■

To convert from Celsius to Fahrenheit, we have used the equation

$$T_{°F} = 1.80\,(T_{°C}) + 32$$

To convert a Fahrenheit temperature to Celsius, we need to rearrange this equation to isolate Celsius degrees ($T_{°C}$). Remember, we can always do the same operation to both sides of the equation. First subtract 32 from each side:

$$T_{°F} - 32 = 1.80\,(T_{°C}) + \underbrace{32 - 32}_{\text{Sum is zero}}$$

to give

$$T_{°F} - 32 = 1.80(T_{°C})$$

Next divide both sides by 1.80

$$\frac{T_{°F} - 32}{1.80} = \frac{\cancel{1.80}(T_{°C})}{\cancel{1.80}}$$

to give

$$\frac{T_{°F} - 32}{1.80} = T_{°C}$$

or

$$\underbrace{T_{°C}}_{\text{Temperature in °C}} = \frac{\overbrace{T_{°F}}^{\text{Temperature in °F}} - 32}{1.80}$$

$$T_{°C} = \frac{T_{°F} - 32}{1.80}$$

EXAMPLE 2.12 Temperature Conversion: Fahrenheit to Celsius

One of the body's responses to an infection or injury is to elevate its temperature. A certain flu victim has a body temperature of 101 °F. What is this temperature on the Celsius scale?

SOLUTION

The problem is 101 °F = ? °C. Using the formula

$$T_{°C} = \frac{T_{°F} - 32}{1.80}$$

yields

$$T_{°C} = ?\,°C = \frac{\overset{T_{°F}}{101} - 32}{1.80} = \frac{69}{1.80} = 38$$

That is, 101 °F = 38 °C.

Self-Check **EXERCISE 2.8** An antifreeze solution in a car's radiator boils at 239 °F. What is this temperature on the Celsius scale?

See Problems 2.75 through 2.78.

In doing temperature conversions, you will need the following formulas.

Temperature Conversion Formulas

- Celsius to Kelvin $T_K = T_{°C} + 273$
- Kelvin to Celsius $T_{°C} = T_K - 273$
- Celsius to Fahrenheit $T_{°F} = 1.80(T_{°C}) + 32$
- Fahrenheit to Celsius $T_{°C} = \dfrac{T_{°F} - 32}{1.80}$

2.8 Density

OBJECTIVE: To define density and its units.

Lead has a greater density than feathers.

When you were in elementary school, you may have been embarrassed by your answer to the question "Which is heavier, a pound of lead or a pound of feathers?" If you said lead, you were undoubtedly thinking about density, not mass. **Density** can be defined as the amount of matter present *in a given volume* of substance. That is, density is mass per unit volume, the ratio of the mass of an object to its volume:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

It takes a much bigger volume to make a pound of feathers than to make a pound of lead. This is because lead has a much greater mass per unit volume—a greater density.

The density of a liquid can be determined easily by weighing a known volume of the substance as illustrated in Example 2.13.

EXAMPLE 2.13 Calculating Density

Suppose a student finds that 23.50 mL of a certain liquid weighs 35.062 g. What is the density of this liquid?

SOLUTION

We can calculate the density of this liquid simply by applying the definition

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{35.062 \text{ g}}{23.50 \text{ mL}} = 1.492 \text{ g/mL}$$

This result could also be expressed as 1.492 g/cm^3 because 1 mL = 1 cm^3. ■

The volume of a solid object is often determined indirectly by submerging it in water and measuring the volume of water displaced. In fact, this is the most accurate method for measuring a person's percent body fat. The person is submerged momentarily in a tank of water, and the increase in volume is measured (see Figure 2.10). It is possible to calculate the body density by using the person's weight (mass) and the volume of the person's body determined by submersion. Fat, muscle, and bone have different densities (fat is less dense than muscle tissue, for example), so the fraction of the person's body that is fat can be calculated. The more muscle and the less fat a person has, the higher his or her body density. For example, a muscular person weighing 150 lb has a smaller body volume (and thus a higher density) than a fat person weighing 150 lb.

EXAMPLE 2.14 Determining Density

The most common units for density are g/mL = g/cm^3.

At a local pawn shop a student finds a medallion that the shop owner insists is pure platinum. However, the student suspects that the medallion may actually be silver and thus much less valuable. The student buys the medallion only after the shop owner agrees to refund the price if the medallion is

Figure 2.10

a Tank of water

b Person submerged in the tank, raising the level of the water returned within two days. The student, a chemistry major, then takes the medallion to her lab and measures its density as follows. She first weighs the medallion and finds its mass to be 55.64 g. She then places some water in a graduated cylinder and reads the volume as 75.2 mL. Next she drops the medallion into the cylinder and reads the new volume as 77.8 mL. Is the medallion platinum (density = 21.4 g/cm^3) or silver (density = 10.5 g/cm^3)?

SOLUTION

The densities of platinum and silver differ so much that the measured density of the medallion will show which metal is present. Because by definition

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

to calculate the density of the medallion, we need its mass and its volume. The mass of the medallion is 55.64 g. The volume of the medallion can be obtained by taking the difference between the volume readings of the water in the graduated cylinder before and after the medallion was added.

$$\text{Volume of medallion} = 77.8\text{ mL} - 75.2\text{ mL} = 2.6\text{ mL}$$

The volume appeared to increase by 2.6 mL when the medallion was added, so 2.6 mL represents the volume of the medallion. Now we can use the measured mass and volume of the medallion to determine its density:

$$\text{Density of medallion} = \frac{\text{mass}}{\text{volume}} = \frac{55.64\text{ g}}{2.6\text{ mL}} = 21\text{ g/mL}$$

or

$$= 21\text{ g/cm}^3$$

The medallion is really platinum.

Self-Check EXERCISE 2.9 A student wants to identify the main component in a commercial liquid cleaner. He finds that 35.8 mL of the cleaner weighs 28.1 g. Of the following possibilities, which is the main component of the cleaner?

Substance	Density, g/cm^3
chloroform	1.483
diethyl ether	0.714
isopropyl alcohol	0.785
toluene	0.867

See Problems 2.89 and 2.90. ■

EXAMPLE 2.15 Using Density in Calculations

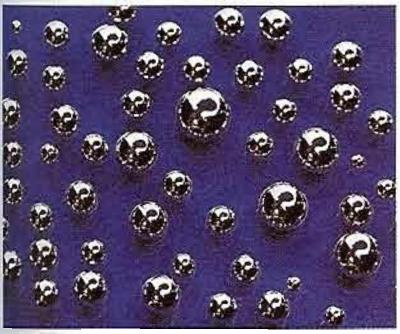
Mercury has a density of 13.6 g/mL. What volume of mercury must be taken to obtain 225 g of the metal?

SOLUTION

To solve this problem, start with the definition of density,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

and then rearrange this equation to isolate the required quantity. In this case we want to find the volume. Remember that we maintain an equality



Spherical droplets of mercury, a very dense liquid.

when we do the same thing to both sides. For example, if we multiply *both sides* of the density definition by volume,

$$\text{Volume} \times \text{density} = \frac{\text{mass}}{\cancel{\text{volume}}} \times \cancel{\text{volume}}$$

volume cancels on the right, leaving

$$\text{Volume} \times \text{density} = \text{mass}$$

We want the volume, so we now divide both sides by density,

$$\frac{\text{Volume} \times \cancel{\text{density}}}{\cancel{\text{density}}} = \frac{\text{mass}}{\text{density}}$$

to give

$$\text{Volume} = \frac{\text{mass}}{\text{density}}$$

Now we can solve the problem by substituting the given numbers:

$$\text{Volume} = \frac{225 \text{ g}}{13.6 \text{ g/mL}} = 16.5 \text{ mL}$$

We must take 16.5 mL of mercury to obtain an amount that has a mass of 225 g. ■

The densities of various common substances are given in Table 2.8. Besides being a tool for the identification of substances, density has many other uses. For example, the liquid in your car's lead storage battery (a solution of sulfuric acid) changes density because the sulfuric acid is consumed as the battery discharges. In a fully charged battery, the density of the solution is about 1.30 g/cm^3. When the density falls below 1.20 g/cm^3, the battery has to be recharged. Density measurement is also used to determine the amount of antifreeze, and thus the level of protection against freezing, in the cooling system of a car. Water and antifreeze have different densities, so the measured density of the mixture tells us how much of each is present. The device used to test the density of the solution—a hydrometer—is shown in Figure 2.11.

Figure 2.11
A hydrometer being used to determine the density of the antifreeze solution in a car's radiator.

Table 2.8 Densities of Various Common Substances at 20 °C

Substance	Physical State	Density (g/cm^3)
oxygen	gas	0.00133*
hydrogen	gas	0.000084*
ethanol	liquid	0.785
benzene	liquid	0.880
water	liquid	1.000
magnesium	solid	1.74
salt (sodium chloride)	solid	2.16
aluminum	solid	2.70
iron	solid	7.87
copper	solid	8.96
silver	solid	10.5
lead	solid	11.34
mercury	liquid	13.6
gold	solid	19.32

*At 1 atmosphere pressure