A Significant Review

Let's start off with scientific notation…

Large numbers (numbers for which the absolute value is greater than 1) will always have a positive exponent when in scientific notation. When converting to scientific notation, you move the decimal point until there is a single digit to the left. The number of places that the decimal spot moved becomes the exponent and the "x10".

Example: -450000 → $-4.5x10^5$. The decimal point was moved 5 times to the left, so the exponent is 5.
Example: 106709001 → $1.06709001x10^8$. The decimal was moved to the left by 8 spots so the exponent is 8.
Example: 57293.264 → $5.7293264x10^4$ since the decimal was moved 4 times to the left.

Small numbers (numbers between 1 and -1) will always have a negative exponent when in scientific notation. When converting to scientific notation, you move the decimal point until there is a single digit to the left. The number of places that the decimal spot moved becomes the exponent and the "$x10^-$".

Example: 0.0003528 → $3.528x10^{-4}$. The decimal moved 4 times to the right, so the exponent become -4.
Example: -0.0000000000000058500 → $-5.8500x10^{-15}$. The decimal point was moved 15 times to the right, so the exponent became -15.
Example: 0.002 → $2x10^{-3}$ since the decimal was moved 3 times to the right.

You try:

1a) 54,670,000,000
1b) -5526.7
1c) 0.03289
1d) 100.00
1e) -0.000093740
1f) 9999.606
1g) 2800
1h) -0.00000005883
1i) 0.00008
1j) 0.11250

How many significant figures in a number:

First and foremost, you need to be able to tell how many sig. figs. are in a number. Here is a recap of the 3 rules I gave you:

1) If the number is in scientific notation:
The number of digits shown is equal to the number of sig. figs.

Examples:

$6.626x10^{-34}$ has **4** significant figures ($6.626x10^{-34}$)
$8.30x10^4$ has **3** significant figures ($8.30x10^4$)
$3.0x10^1$ has **2** sig. figs. ($3.0x10^1$)

2) If the number has a decimal in it:
Start at the RIGHT of the number and count to the left until you get to the last NONZERO number, this is the number of sig. figs. Any additional zeros to the left are NOT significant.

Examples:

195.3040 has **7** sig. figs. (195.3040)
0.003081 has **4** sig. Figs. (0.003081)
180048.00 has **8** sig. figs. (180048.00)
0.0000002 has 1 sig. fig. (0.0000002)
10. has 2 sig. fig. (10.)

3) If the number does NOT have a decimal in it:
Start at the LEFT of the number and count to the right until you get to the last NONZERO number, this is the number of sig. figs.

Examples:

160 has **2** sig. figs. (160)
20000 has **1** sig. figs (20000)
704 has **3** sig. figs. (704)
49003100 has **6** sig. figs. (49003100)
10 has 1 sig. fig. (10)

You try:
2a) 6200
2b) 1.032
2c) 420.
2d) 3.750×10^{-6}
2e) 0.0006000
2f) 1×10^{4}
2g) 35000000
2h) 23.4400
2i) 100.0003
2j) 100.

Significant figures in calculations

There are two distinct rules that you need to be able to use and keep straight.

Addition and/or subtraction:

The rule for addition and subtraction is based on the precision of the values being added and/or subtracted. In simpler terms, you need to count the number of decimal places in each of the values. The answer must have the same number of decimal places as the value in the problem that has the ***FEWEST*** **DECIMAL PLACES**. What does that mean? If you add 5.12345 (5 decimal places), 12.123 (3 decimal places), and 0.12 (2 decimal places), your answer must have 2 decimal places.

Example: $2500.0 + 1.236 + 367.01$

First, write the digits vertically with the decimal points **lined up** and find the number of decimal places for each value (this will help until you get more comfortable with the process). The answer must have the same number of decimal spots as the value in the problem with the fewest decimal places.

$$\begin{array}{r} 2500.0 \\ +\quad 1.236 \\ +\ 367.01 \\ \hline \end{array}$$

2500.0 has one decimal spot, 1.236 has three decimal spots and 367.01 has two decimal spots, therefore the answer must have one decimal spot. You add them up and then round as follows:

$$\begin{array}{r} 2500.0 \\ +\quad 1.236 \\ +\ 367.01 \\ \hline 2868.246 \end{array}$$

The number of decimal places allowed in the answer is dictated by the first value (because that value has the fewest decimal places), so you must round to that digit (the 2 in 2868.246 here). The answer is **2868.2**.

Example: $0.007560 + 0.0133$

$$\begin{array}{r} 0.007560 \\ +\ 0.0133 \\ \hline 0.020860 \end{array}$$

So the answer is 0.0209

Example: $0.01 - 0.006125$

$$\begin{array}{r} 0.01 \\ -\ 0.006125 \\ \hline 0.003875 \end{array}$$

You can only 2 decimal places, so the answer is 0.00

Example: $0.0417 + 0.956 + 0.0022954$

$$\begin{array}{r} 0.0417 \\ +\ 0.956 \\ +\ 0.0022954 \\ \hline 0.9999954 \end{array}$$

Again, your answer must have the same number of decimal spots as the value with the fewest in the question; 3 in this case, so the answer is **10.000**

Multiplication and/or division:

The rule for multiplication and division is all about how many sig. figs. a number has. The value in the calculation that has the ***FEWEST*** number of **SIGNIFICANT FIGURES** determines the number of sig. figs. in your answer. If you are multiplying 3 different numbers, one has 4 s.f., one has **2** s.f. and one has 7 s.f., your answer can only have **2** s.f.

Example: 0.01116 x 23.44600 = 0.26165736
0.01116 has 4 s.f. and 23.44600 has 7 s.f. Therefore the answer is limited to 4 s.f.
The answer would be rounded to **0.2617**

Example: 26.375 x 3791 = 99987.625
26.375 has 5 s.f. and 3791 has 4 s.f., so the answer is again limited to 4 s.f. This is a fairly large number, so put it into scientific notation before rounding. It becomes 9.9987625×10^4. Now do your rounding and you get 10.00×10^4. There can only be one digit to the left of the decimal, so the final answer is **1.000×10^5**.

Example: $\frac{3.14159}{502000} = 0.000006258$
3.14159 has 6 s.f. and 502000 has 3 s.f. so the answer can only have 3 s.f. The answer is **0.00000626** or **6.26×10^{-6}**

Examples: $\frac{536 \times 0.3301 \times 60.002}{0.0048 \times 12.1} = 182788.73738$
536 has 3 s.f., 0.3301 has 4 s.f., 60.002 has 5 s.f., 0.0048 has **2** s.f., and 12.1 has 3 s.f., so the answer can only have **2** s.f. This is a large number, so put it into scientific notation BEFORE rounding → 1.8278873738×10^5. Since you can only keep 2 s.f., the answer is **1.8×10^5**.

You try:
3a) $160 \times 0.3490 \times 23.1$
3b) $2.3806 + 0.01$
3c) $\frac{0.2689}{0.000159}$
3d) $11.3 - 2$

3e) $1500. \div 25$

3f) $3.65 \times 10^{-3} \times 9.822 \times 10^{4}$

3g) $\dfrac{2.21100 \times 10^{2}}{32.1 \times 0.002000}$

3h) $0.34864 + 1$

3i) $26.1 - .00030000$

3j) $1200 + 49.49 + 1.004$

3k) 33.3×3.0

Mixed operations – multiplication/division AND addition/subtraction in the same problem:

When working with significant figures where there is a mixture of operations, the rules for the individual operations do not change, but the order in which those operations are performed is important. The order in which you perform the calculations follows the “order of operations” which you may remember from algebra. That order is: **p**arentheses, **e**xponents, **m**ultiplication, **d**ivision, **a**ddition, and **s**ubtraction (**p**lease **e**xcuse **m**y **d**ear **a**unt **s**ally). After each of these steps, you need to mark the last significant figure you are allowed in that step (usually with a line over that digit) so that you can keep track of what the limiting significant figure is in each step. You do NOT want to round your answer after each step, but rather you should wait to do the rounding at the end of the entire problem and this is why it is important to mark the last sig. fig allowed in each step. I am going to start these examples with something we have already seen this semester: isotopic abundance calculations.

Example: Gallium has two stable isotopes, gallium-69 and galium-71. If the mass of gallium-69 is 68.926 *amu* and the mass of galium-71 is 70.9247 *amu*, then what are the percent abundances of each isotope?

The beginning equation is: 68.926 X + 70.9247 • (1 - X) = 69.72

According to the order of operations, we need to clear the parentheses first, but since we don’t know what X is, there is nothing we can do here. The first operation we are actually going to do is the multiplication step. The equation becomes:

68.926 X + 70.9247 - 70.9247 X = 69.72

Because you are multiplying by 1 (an exact number), there is no change in sig. figs. to worry about in this step. Now that all of the multiplication is taken care of, we will deal with subtraction.

$$\begin{array}{lr} 68.926\ X + 70.9247 - 70.9247\ X = 69.72 \\ \quad - 70.9247 \qquad\qquad - 70.9247 \\ \hline 68.926\ X \qquad - 70.9247\ X = -1.2\bar{0}47 \end{array}$$

Notice the line over the top of the zero on the right hand side of the equation. Since we are subtracting, we base our answer on the number of **decimal places** in the values we are subtracting. 69.72 has 2 decimal places and 70.9247 has 4. This means that my answer must have 2 decimal places and I indicate that with the line over the second decimal place in the -1.2047. The next step is to perform the subtraction on the left side of the equation.

$$68.926\ X - 70.9247\ X = -1.2\bar{0}47$$

$$-1.99\bar{8}7\ X = -1.2\bar{0}47$$

Again following the rules for addition/subtraction, I have placed a line over 8 in the value on the left because we are only allowed 3 decimal places after performing this subtraction. Also note that I have NOT done any rounding yet! The next step is to divide both sides by -1.9987 in order to get X by itself.

$$\frac{-1.99\bar{8}7\ X}{-1.99\bar{8}7} = \frac{-1.2\bar{0}47}{-1.99\bar{8}7} \implies X = 0.602741782$$

Following the rules for multiplication/division of sig. figs., we must base the sig. figs. in our answer on the number of **significant figures** in values we are dividing. Looking at the lines that we have been placing above our values as we have proceeded, we see that -1.2047 has 3 sig. figs. and -1.9987 has 4 sig. figs. Because of this, the answer is **0.0603**

Example: $(3.86200 + 0.0987) \times 0.1345$

We start with the parentheses and because the operation with the parentheses is addition, we will follow that rule and base our intermediate answer on **decimal places**. The first value has 5 decimal places and the second has 4, so our answer must have 4 and we will denote that by putting a line over the top of the 4th decimal place in the intermediate answer.

$$(3.86200 + 0.0987) \times 0.1345 \Rightarrow 3.960\bar{7}2 \times 0.1345$$

The next step is multiplication, so the answer will be based on the number of **significant figures** in the two values. The first has 5 sig. figs. (we know that because of the line) and the second has 4, so our answer will have 4.

$$3.960\bar{7}2 \times 0.1345 = 0.53271684 \Rightarrow \mathbf{0.5327} \text{ which is the answer}$$

Example: $28.5821 - 0.0777 \times 1.430 \times 10^3$

Remember your order of operations!! We must do the multiplication step first which means the sig. figs in the intermediate answer will be determined by the number of **sig. figs.** in the values being multiplied (3 in 0.0777 and 4 in 1.430×10^3). We'll put a line over the last sig. fig. we are allowed to keep.

$$28.5821 - 0.0777 \times 1.430 \times 10^3 \Rightarrow 28.5821 - 11\bar{1}.111$$

The next step is subtraction which means that the number of sig. figs. in the answer is based on the number of **decimal places** in the values being subtracted (4 in the first value and 0 in the second, look for the line!!)

$$28.5821 - 11\bar{1}.111 = -82.529 \Rightarrow \mathbf{-83} \text{ which is the answer}$$

Example:

$$\frac{(3.21 - 238.0)}{(0.238 + 4.00)}$$

Do each set of parentheses first making sure to mark the last sig. fig. you are allowed to keep (for this question, based of course on the addition/subtraction rules)

$$\frac{(3.21 - 238.0)}{(0.238 + 4.00)} \Rightarrow \frac{(-234.\bar{7}9)}{(4.2\bar{3}8)}$$

The final step is a division, so follow that rule. The top value as 4 sig. figs. and the bottom has 3.

$$\frac{(-234.\bar{7}9)}{(4.2\bar{3}8)} = -55.401 \Rightarrow \mathbf{-55.4} \text{ which is the answer}$$

Example: $$\frac{(62.43 + 46.51286)}{7.91801 \times 10^{-8}}$$

$$\frac{(62.43 + 46.51286)}{7.91801 \times 10^{-8}} \Rightarrow \frac{(108.9\bar{4}286)}{7.91801 \times 10^{-8}}$$

$$\frac{(108.9\bar{4}286)}{7.91801 \times 10^{-8}} \Rightarrow 137587\bar{}1232$$

$$\frac{(108.9\bar{4}286)}{7.91801 \times 10^{-8}} \Rightarrow 1375\bar{8}71232$$

Remember to put large numbers into scientific notation BEFORE rounding

$1375\bar{8}71232 \Rightarrow 1.375\bar{8}71232 \times 10^{9} \Rightarrow \mathbf{1.3759 \times 10^{9}}$ which is the answer

Your turn:

4a) There are two stable isotopes of silver, ${}^{107}_{47}\text{Ag}$ (silver-107) and ${}^{109}_{47}\text{Ag}$ (silver-109). The first isotope has a mass of 106.905 *amu* and the second isotope has a mass of 108.9048 *amu*. What are the percent abundances of the two isotopes?

4b) Rhenium has 2 isotopes, ${}^{185}_{75}\text{Re}$ (184.95297 *amu*) and ${}^{187}_{75}\text{Re}$ (186.956 *amu*). Calculate the percent abundance of each isotope.

4c) Antimony has 2 isotopes, ${}^{121}_{51}\text{Sb}$ (120.903824 amu) and ${}^{123}_{51}\text{Sb}$ (122.904222 amu). Calculate the percent abundance of each isotope.

4d) $0.0521 \times (112.40 + (3.02391 + 24.0224 + 31.9988) \times 3.0000000)$

4e) $5.6629 \times (47.90 + 2.0000000 \times (1.00797 + 15.9994))$

4f) $\dfrac{28.641}{(-2.3154 + 2.1)}$

4g) $\dfrac{(-0.4680 + 135.79) \times 16.0}{(128.42 - 129.226)}$

4h) $\dfrac{(0.8000 - 0.7943)}{0.8000}$

4i) $(0.00748214 \times 5.6237 \times 10^{2}) + (1161 \div 20)$

4j) $(28.621 + 81.993) \times 100.000$

4k) $8640. \times 85.4861 + 17.0 \times 317.65$

<u>Significant figures and defined values versus measured values</u>

All measured values limit the sig figs your answer can have while defined values do not. The questions is, how can you tell? Conversions within a system of measure (i.e. metric to metric or standard to standard) and within the same type of unit (i.e. volume to volume or mass to mass) are defined values! Conversions between different systems (i.e. metric to standard) or conversions between different types of units (i.e. time to length or mass to volume) are considered measured values. What does all of this mean? Here are some examples:

$\dfrac{12\text{ in}}{1\text{ ft}}$ is defined because inches and feet are both in the same system of measure (standard) and are both length units. This conversion would not affect the sig figs in the answer.

$\dfrac{1\text{ mile}}{1609\text{ m}}$ is measured because miles are standard units of length while meters are metric units of length. This conversion WOULD affect the sig figs in the answer. (4 sig figs)

$\frac{13.2\ g}{1\ mL}$ is a measured value because grams are a unit of mass and mL are a unit of volume. This conversion WOULD affect the sig figs in your answer. (3 sig figs.)

$\frac{1 \times 10^{-3}\ L}{1\ mL}$ is defined because L and mL are both in the same system of measure (metric) and are both length volume. This conversion would not affect the sig figs in the answer.

Do all of the practice problems. I will send out a key soon, but you should try them without the key first since you won't have a key when you take the exam. When you do get the key; check your answers. If you missed any, don't say "Oh well, missed that one". Look and find out what you did wrong! Good luck!