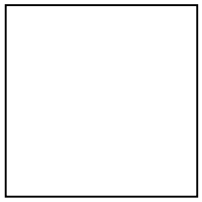
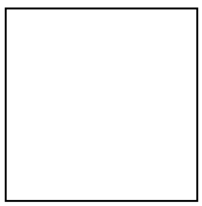
Math 1 **5.5 Irrational Numbers**

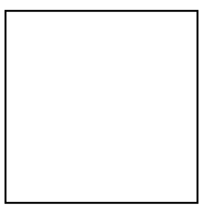


Suppose we have a square whose area is 1. Do we know the length of one side?

Suppose we have a square whose area is 4. Do we know the length of one side?



Suppose we have a square whose area is 5. Do we know the length of one side?



Definition:

The **square root** of a non-negative number *n* is a number such that its square is equal to *n*. that is, the positive square root of *n*, denoted by $\sqrt{n}$, is defined as that number for which $\sqrt{n} \cdot \sqrt{n} = n$

$\sqrt{9} \cdot \sqrt{9} = 9$ but we also know that 3 · 3 = 9, so it seems that $\sqrt{9}$ = 3. Now it is true that (– 3) (– 3) = 9 but mathematicians have agreed that the $\sqrt{\ }$ symbol will denote only positive numbers. Thus, $\sqrt{9} = 3$ and $-\sqrt{9} = -3$

Sometimes square roots are rational numbers.

$\sqrt{4}$

$\sqrt{25}$

$\sqrt{81}$

$\sqrt{\frac{36}{49}}$

Is $\sqrt{2}$ a rational number? That is, is there a reduced rational number, $\frac{a}{b}$, such that $\frac{a}{b} \cdot \frac{a}{b} = 2$?

The square root of any number that is not a perfect square is an irrational number.

However, even when a number is irrational, we can sometimes simplify it.

For example, $\sqrt{12}$ is not a perfect square, but $\sqrt{12} = \sqrt{4 \cdot 3}$ and 4 is a perfect square. So, we can rewrite $\sqrt{12}$ as $\sqrt{4} \cdot \sqrt{3}$

Thus, $\sqrt{12}$ = __________

This is called "simplifying a square root."

In general, a square root is simplified when:

1. the number under the square root sign has no exponent larger than 2.
2. there is no fraction under the square root sign (and no negative exponents)
3. there is no square root in the denominator of a fraction.

Examples:

$\sqrt{48} =$

$\frac{\sqrt{20}}{\sqrt{4}} =$

$\sqrt{\frac{3}{5}} =$

$\sqrt{3^2 - 4(2)(-3)} =$

$\frac{3 + 6\sqrt{5}}{3} =$

$\sqrt{a^2 + b^2} =$

$\sqrt{(a+b)^2} =$

Pythagorean Theorem:

Pythagoras (569 – 475 BC) began a school and society.

His followers were called Pythagoreans and they studied music, astronomy, geometry, philosophy, and number properties. They had a very secret society.

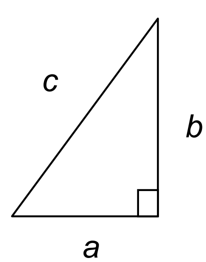
It was considered wrong for a member of this society to claim discovery of any idea himself—any new discoveries were all attributed to the founder, Pythagoras. So, perhaps it is one of the followers who really came up with this theorem.

Years before Pythagoras, the Egyptians and Chinese understood this relationship of the sides of a right triangle, but didn't attribute it to Pythagoras, of course, and didn't call it the Pythagorean Theorem.

It works *only* for right triangles (a right triangle has one 90 degree angle).

Pythagorean Theorem:

For a right triangle with sides *a* and *b* and a hypotenuse *c*, $a^2 + b^2 = c^2$



Also, if $a^2 + b^2 = c^2$ for a triangle with sides of *a*, *b*, and *c*, then the triangle is a right triangle.

Remember: the hypotenuse is always the longest side and is always opposite the right angle.

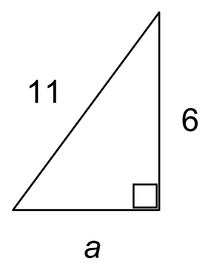
Example: Is a triangle with sides of 9, 15, and 12 a right triangle?

Draw the triangle. Use the Pythagorean Theorem.

Example:

A right triangle has sides of length *a*, and 6, and a hypotenuse of 11. What is the length of the missing side?

1. Draw the triangle and label the sides.



2. Use the Pythagorean Theorem to find the missing side.

Example: A 13 foot ladder leans against a building so that its base is 5 feet from the building. How high up the building does the ladder reach?

1. Draw the picture.

2. Use the Pythagorean Theorem.

Example: There is a pole that needs two guy wires to stabilize it. The guy wires are attached 20 feet from the pole and 30 feet up the pole.

a) What is the exact length of each guy wire?

b) What is the length to the nearest foot?

c) If three guy wires are used, how much wire will be needed (to the nearest foot)?

1. Draw the picture.

2. Use the Pythagorean Theorem.

3. Answer the questions.