Math 1 5.3 Integers

So far we have been looking mostly at Natural Numbers and Whole Numbers.

The set of integers is the set containing all the whole numbers {0, 1, 2, 3, …} and all the opposites of the Natural Numbers {-1, -2, -3, …} So the entire set of integers is {… – 3, – 2, – 1, 0, 1, 2, 3, 4, …} and is often denoted by **Z**.

A quick review of arithmetic with integers…

Adding: if the signs are the same, combine the numbers and use the common sign.

if the signs are different, find the difference of the numbers and use the sign of the "bigger" one.

Examples: $4 + 5 =$

$-5 + -7 =$

$6 + -10 =$

Subtracting: algebraically add the opposite.
In other words, change the subtracting sign to + and change the sign of the number being subtracted.

Examples: $16 - 5 = 16 + (-5)$

$27 - (-5) =$

$-8 - (-5) =$

Multiplying & Dividing: an even number of negative factors yields a positive product/quotient.

an odd number of negative factors yields a negative product/quotient

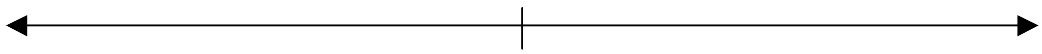
Examples: $(-3)(4) =$

$(-1)(-1)(-1)(-1)(-1)(-1) =$

$(-2)(-3)(-2)(-3)(-2) =$

$\frac{-28}{7} =$

Absolute Value: the distance a number is from zero.



With the Integers we have the properties we had with the Whole Numbers:

Closure

Commutative properties of Addition and Multiplication

Associative properties of Addition and Multiplication

Distributive property of Multiplication over Addition

In fact there are 2 more properties of the Whole Numbers we haven't talked about yet.

Additive Identity:

Multiplication Identity:

But with the integers we also get a new property…

The **Additive Inverse**: For any integer A there is an integer – A such that
$A + (-A) = 0$ and $(-A) + A = 0$

Is the set of Integers closed under Subtraction?

Is the set of Integers closed under Division?

Math 1 5.4 Rational Numbers

The set of rational numbers is the first set where it is impractical to use the roster method of listing all the elements. Recall that we described the set of rational numbers as:

$$\mathbf{Q} = \left\{ \frac{a}{b} \;\middle|\; a \in \mathbf{Z},\ b \in \mathbf{Z},\ b \neq 0 \right\}$$

Some terms associated with Rational Numbers:

numerator: the “top” number, in the definition *a*

denominator: the “bottom” number, in the definition *b*

proper fraction: when $a < b$

improper fraction: when $a > b$

whole number: when $b \mid a$

reduced fraction: when *a* and *b* have no common factors other than 1

Operations with fractions:

Making equivalent fractions: (using the Multiplication Identity)

$$\frac{a}{b} \cdot \frac{k}{k} =$$

Example: $\frac{4}{7}$

Adding and Subtracting fractions: MUST have common denominators first, then add/subtract the numerators.

Examples: $\frac{5}{24} - \frac{7}{30} =$ use what we know of prime factors to find LCM

Multiplying fractions: multiply numerators, multiply denominators, reduce the product.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Examples: $\frac{2}{5} \cdot \frac{3}{7} =$

$\frac{6}{35} \cdot \frac{21}{20} =$

Dividing fractions: multiply by the reciprocal of the second term.
(change the divide sign to multiply and "flip" the second fraction)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Examples: $\frac{-3}{4} \div \frac{2}{7} =$

$1\frac{2}{3} \div 2\frac{4}{9} =$

Rational numbers have the same properties as the Integers:

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In addition, with the set of Rational Numbers we also have:

Multiplication Inverse: For any non-zero rational number, $\frac{a}{b}$, there is a rational number, $\frac{b}{a}$, such that $\frac{a}{b} \cdot \frac{b}{a} = 1$

Example:

The set of Rational Numbers also has a property called **density**. We say the set of Rational Numbers is **dense**.

Density means that for any 2 rational numbers there can always be found a rational number between them. In other words, two rational numbers, $\frac{a}{b}$ and $\frac{c}{d}$ where $\frac{a}{b}$ is less than $\frac{c}{d}$, there exists a rational number, $\frac{m}{k}$, where $\frac{a}{b} < \frac{m}{k} < \frac{c}{d}$.

Example: $\frac{1}{2}$ $\frac{5}{6}$