The most basic set of numbers is called the set of Counting Numbers or the set of Natural Numbers. We saw in Chapter 2 that this set is denoted by **N** and looks like {1, 2, 3, 4, 5, 6, . . . }

With this set of numbers, **N**, and an operation, like **addition**, we find that there are properties that always hold. First we need to define addition.

Addition: remembering our set theory from Chapter 2 helps us with the definition of addition.

Let A and B be *disjoint* sets and let n(A) = a and n(B) = b. Then,

a + b = n(A U B)

Example 1: Let A = Let B =

Then n(A) = and n(B) =

A U B = { and n(A U B) =

This shows that _____ + ______ = _______

What happens if A and B are <u>not</u> *disjoint* sets?

Example 2:

Let A = and let B =

Then, n(A) = and n(B) =

Here, A U B = and n(A U B) =

but _______ + _______ ≠ _________ Why????

The **Closure Property** of the Natural Numbers under Addition:

(Notice that we need both a set and an operation)

For any two natural numbers, A and B, their sum (A + B) is also a natural number and that sum is unique to A and B.

Other phrases that mean the same:

"The set of Natural numbers is closed under addition."

"The Closure Property holds for the set of Natural numbers with addition."

Example: Is the set A = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} closed under addition?

How do you know?

This is an example of a proof by counter-example.

The **Commutative Property** of the set of Natural numbers under addition states that…

For any two natural numbers, C and D, C + D yields the same sum as D + C.

Or, $C + D = D + C$

In other words,

The **Associative Property** for the set of Natural numbers under addition states that…

For any natural numbers A, B, and C it is always true that (A + B) + C yields the same sum as A + (B + C).

Or, $(A + B) + C = A + (B + C)$

In other words,

Often, these two properties are used together to make adding easier. For example, finding this sum:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = ??$

is easier if we use the Commutative and Associative Properties to rearrange the numbers like this…

( ) + ( ) + ( ) + ( ) + 5 = ____________

Addition is not the only operation that we can apply to the Natural numbers in order to have properties hold.

Multiplication is defined as follows:

$n \times a = a + a + a + \ldots + a$

(n is the number of times we add a to itself)

So, 3 x 5 =

while 5 x 3 =

What does 0 x a = ?

What does n x 0 = ?

Does the **Closure** property hold for multiplication of Natural numbers?

Do the Natural numbers have the **Commutative property** for multiplication?

Do the Natural numbers have the **Associative property** for multiplication?

There is a very important property of the Natural numbers that uses both operations.

The **Distributive Property of Multiplication over Addition** uses both adding and multiplying. If fact, it is the only property that requires two operations!

Distributive Property of Multiplication over Addition: For natural numbers A, B, and C it is always true that $A \cdot (B + C)$ yields the same natural number as $A \cdot B + A \cdot C$.

That is, $A \cdot (B + C) = A \cdot B + A \cdot C$ another way to write it: $A(B + C) = AB + AC$

This also says that $AB + AC = A(B + C)$

Example A:

Why is this property so important?

Making arithmetic easier: example. 7 x 186

in Algebra: example 1. $7x^3\left(4x^2 + 5x\right) =$

example 2. $36y^3 + 45y^2 =$

Can we switch the adding and multiplying? That is, does $A + (BC) = (A + B)(B + C)$??

Let's use our Example A numbers again.

Subtraction. Strange as it seems, subtraction is defined by addition.

For numbers A and B, $A - B = x$ means that $A = B + x$

Example:

Is the set of Natural numbers **closed** under subtraction?

Is subtraction **commutative** in the Natural numbers?

Is subtraction **associative** in the Natural numbers?

We can also look at the properties of a set of numbers or objects even when the operation is not our standard addition, subtraction, multiplication, or division.

Consider this set and operation in table format:

⊕		Δ	Ω
	Ω		Δ
Δ		Δ	Ω
Ω	Δ		Ω

1. Find ⊕ Δ.

2. Find Δ ⊕ Ω.

3. Does Ω ⊕ = ⊕ Ω?

4. Is this set **commutative** for ⊕? How do you know?

5. Does (Ω ⊕ Δ) ⊕ Δ = Ω ⊕ (Δ ⊕ Δ) ?