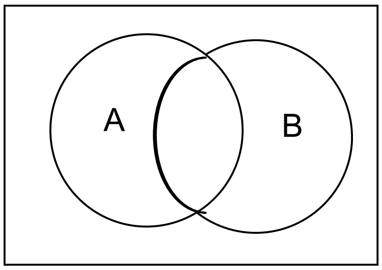
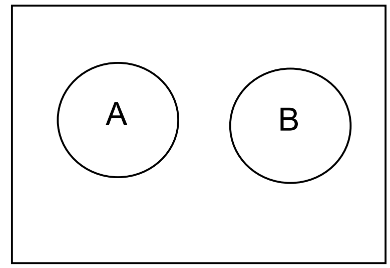
We noticed in the previous section that when sets share elements, those elements are in an overlap section in the Venn Diagram. We generally draw the Venn Diagram with an overlap even if it turns out to be empty.



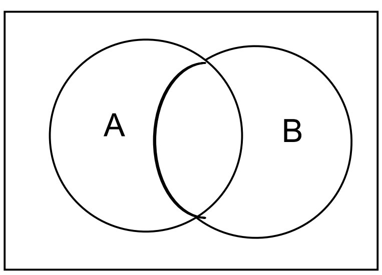
If the sets had No elements in common, the Venn Diagram could have been drawn this way.



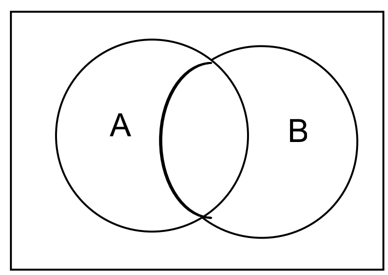
The set of elements common to both set A and set B is a SET called the ______________________ of sets A and B and is denoted by $A \cap B$.

The set of elements in either A or B (or both) is a SET called the _______________ of sets A and B and is denoted by $A \cup B$.

In a Venn Diagram we can use shading to indicate these two sets.



$A \cap B$



$A \cup B$

Example: A = {1, 2, 3, x, y, z} B = {2, 4, 6, 8, w, x, y} C = {8, 9, a, b}

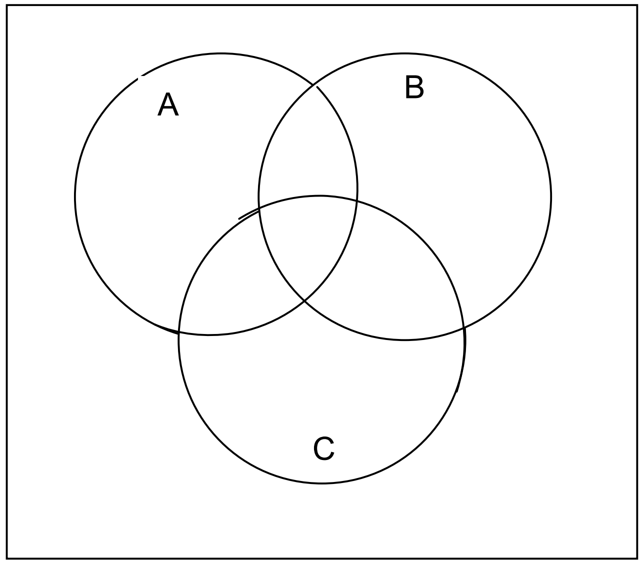
Find $A \cap B$

Find $A \cup B$

Find $A \cap C$

We can use Venn Diagrams with 2 or 3 sets but beyond that they get too complicated to be reasonable.

Example: U = { 1, 2, 3, 4, 5, 6, 7, 8, 9}   A = {2, 4, 6, 8}   B = {1, 3, 5, 7}   C = {5, 7}



Fill in the Venn Diagram and then find the following:

1. A U C

2. B U C

3. B ⋂ C

4. A ⋂ C

5. A ⋂ B ⋂ C

6. A’

7. C’

The cardinality of the intersection of sets is found by counting the number of elements in ______________________________.

The cardinality of the union of sets is a bit more involved because there is overlap in the intersection. If we are looking at the Venn Diagram, you can just add the elements you see. (In the above example, n(A U B U C) is 4 + 2 + 2 = 8) However, if we added the number of elements in A and B and C [n(A) + n(B) + n(C)= 4 + 4 + 2 = 10….not 8]. This is because some elements are in both B and C.

For any 2 sets A and B, $n(A \cup B) = n(A) + n(B) - (A \cap B)$

Example: In a particular school 25 students belonged to the Math Club and 16 students belonged to the Science Club. If 7 students belonged to both clubs, how many different students belonged to these clubs?

Drawing the Venn Diagram we get:

We can see that we need the union of the sets without the overlap to answer the question.

Section 2.3

Now that we understand the intersection, union, and complement of sets, we can use this knowledge for more complicated applications.

Example: For any sets A and B use a Venn Diagram to find the following:

1. $\overline{A} \cup \overline{B}$ 2. $\overline{A \cup B}$ 3. $\overline{A} \cap \overline{B}$

Notice that while the Venn Diagrams of examples ____ and ___ are not the same, the Venn Diagrams of ____ and ____ are the same.

This is called De Morgan's Laws for Sets.

For any two sets, A and B, it is always true that…

$\overline{A \cup B} =$

and

$\overline{A \cap B} =$

Some for you to try: Draw the Venn Diagram and shade appropriately.

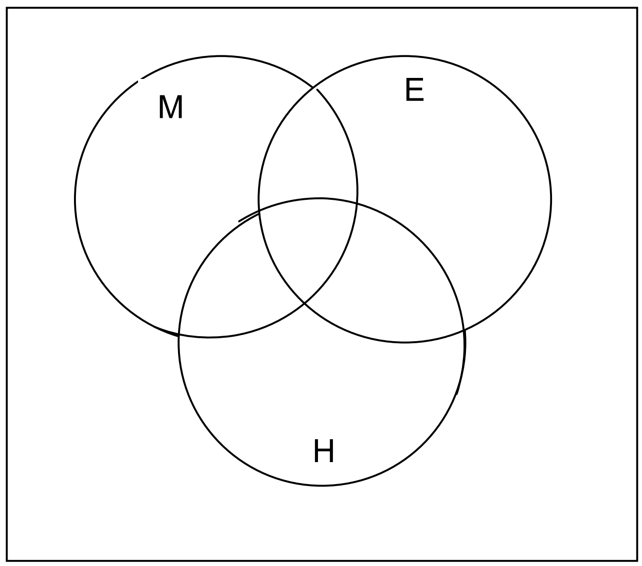
1. $(A \cup B) \cap C$

2. $(A \cup B) \cap C'$

3. $A \cup (B \cap C)$

We can now use Venn Diagrams to analyze survey questions.

A survey of 250 students found that 89 students were taking a math class, 105 students were taking an English class, 87 students were taking a history class, 24 students were taking math and English, 20 students were taking math and history, 17 students were taking English and history, and 11 students were taking all three subjects.



1. How many students were only taking math?

2. How many students were only taking English?

3. How many students were taking only history?

4. How many students were not taking any of these courses?