Sets are a way of sorting objects into groups. Although the word “set” in mathematics is considered an undefined term, we can think of a set as a ________________________.

The objects in a set are called **elements**.

example: consider the set A = {*a, b,* {*c*}*, d* }

a is an element of the set because *a* is found by itself in the set {*a, b,* {*c*}*, d* }.

However, *c* is not an element of the set A = {*a, b,* {*c*}*, d* } because the *c* in the set A is in a set itself. So, {*c*} is an element of the set A, but *c* is not an element of the set A = {*a, b,* {*c*}*, d* }.

What is *c* an element of?

Element notation is mathematical short hand for the English sentence. BE CAREFUL!!

In a **well defined** set it is very clear whether __________________________________.

An example of a well defined set:

An example of a set that is not well defined:

There are several ways to describe sets:

1. Descriptive

   example:

2. Roster

   example:

3. Set Builder Notation

   example:

4. Capital Letters (and special letters)

A or other letter of the alphabet

N

W

I or **Z**

Q

R

C

The set that contains every element under consideration is called the __________________

and is denoted by:

A set that does not contain any elements is called an ________________________ and is

denoted by:

Be careful with notation here.

A one-to-one correspondence between two sets A and B means that __________________

____

____ .

Example: A = {1, 2, 5} B = { x, w, m} C = {4, x, T, 7 }

Two sets which have a one-to-one correspondence are said to be ___________________ and we use the notation: A $\leftrightarrow$ B

Two sets which have exactly the same elements are said to be **equal sets.**

When listing the elements of a set, we do not list duplicates. So the set {1, 3, 2, 1, 1, 3 } is really just the set {1, 2, 3 }. Similarly, a set has infinite copies of each element in it.

The number of distinct elements in a set is called the _________________________ of

the set. Or, sometimes we say the ___________________ number of the set. If A is a set, then n(A) is used to denote the number of distinct elements in set A. Your book uses the notation-- $|A|$ WebAssign may use this as well, but I prefer n(A).

Some problems for you to try:

1. Is the object on the left an element of the set on the right? Add $\in$ or $\notin$ to make a true statement. Can you justify your decisions?

   a) $\{\pi, \varnothing\}$ A = $\{0, 1, \{\varnothing\}, \pi\}$

   b) $\{\pi\}$ B $= \{0, 2, \{2\}, \{\pi\}, 3\}$

   c) 0 C = $\varnothing$

   d) $\varnothing$ D = 0

2. If possible, put each pair of sets into a one-to-one correspondence. Show your arrows.

   a) A = {R, T, W, G}

      B = {4, 5, 7, 9}

   b) G = {2, 4, 6, 8, . . .}

      H = {2, 4, 6, 8 }

   c) J = {1, 2, 3, 4, 5, 6, 7, 8, 9, . . . }

      K = { . . . – 3, – 2, – 1, 0, 1, 2, 3, . . . }

3. What is the cardinality of each set?

a) $A = \{4, 3, 2, 7, 3, 0, \varnothing, \{3, 4, 2\}, \{\varnothing\}, 2\}$

b) $B = \{1, 0, 1, 11, \varnothing, \{0, 1\}, \{ \}, \{1, 1, 0, 11\}, \varnothing\}$

4. Consider the following sets. Determine which are equivalent and which are equal.

$A = \{a, b, c, d\}$ $B = \{W, X, Y, Z\}$ $C = \{c, d, a, b\}$

$D = \{x \mid 0 < x < 5, x \in \mathbf{N}\}$ $E = \varnothing$ $F = \{\varnothing\}$ $G = \{ 0 \}$

$H = \{ \}$ $J = \{x \mid x = 2n + 1$, where n is a whole number$\}$

$K = \{x \mid x = 2w - 1$, where w is a natural number$\}$

Equivalent sets:

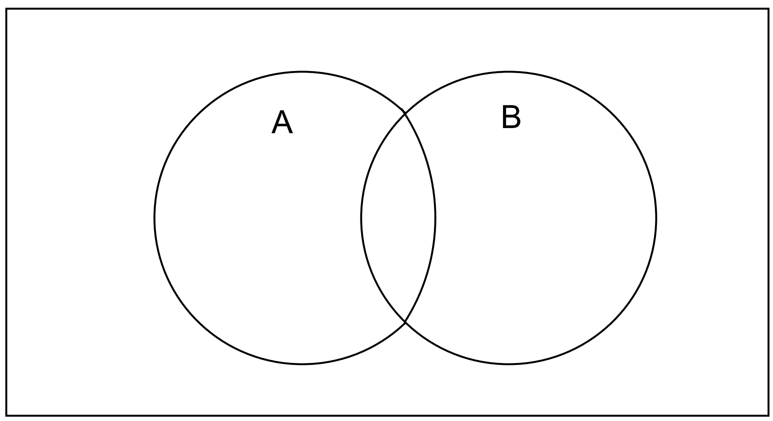
Equal sets:

Venn Diagrams

Venn diagrams are a way to picture sets. Usually, circles are used, but any shape can be used. They are named after John Venn (1834 – 1923) who first used them in a general way but he was not the first to use circles to depict sets.

Venn Diagrams use a rectangle to represent the Universe (i.e. the Universal set). A circle within the rectangle represents a set. Each set described is represented by a circle in the universe.

Example: U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8, 10}
B = {3, 6, 9} Notice that A and B share the element 6



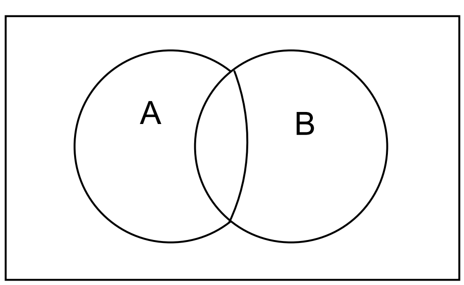
We need to account for all the elements in the Universe while putting the elements of A and B in their respective circles. Where the circles overlap is where any elements in BOTH A and B will go.

All the elements in the Universe that are NOT in a particular set make up a set called the

_________________________ of the set. The text denotes this by a bar over the set name. So, if we want the set whose elements are not in set A, the notation is: $\overline{A}$
Other common notations are A' and A^c.

Once the Venn Diagram is drawn, we can easily find A'. A' = _____________________

To draw A' you shade everything in the Universe that is not A.



In the example we found one element in both set A and set B. However, there are other possibilities for A and B.

- Every element of A could be and element in B.

  example: A = B =

  then the Venn Diagram would look like:

  In this case we say A is a **subset** of B

- Every element of B could be an element of A.

  example: A = B =

  then the Venn Diagram would look like:

  In this case we say B is a **subset** of A

- Sets A and B could be equal.

  example: A = B =

  then the Venn Diagram would look like:

  Here A is a **subset** of B AND
  B is a **subset** of A (an alternate definition of equal sets)

- Sets A and B could have no elements in common.

  example: A = B =

  then the Venn Diagram would look like:

  In this case A and B are said to be **disjoint** sets

The symbol for subset is a sideways capital U and half an equal sign.

NOTE: The symbol cannot stand alone!

If A is a subset of B, we write $A \subseteq B$.

If A is not a subset of B, we write $A \not\subseteq B$.

There is a special case when A is a subset of B but B has elements that A does not. Then

we say that A is a _______________ subset of B. This situation is denoted by $A \subset B$, meaning every element of A is an element in B but $A \neq B$.

Consider the following sets. Answer the question and briefly describe your reasoning.

A = {1, 2, 3, 4}   B = {2}   C = { }   D = {1, {2}, 3, 4}   E = {1, 2, 4}

1. Is $B \subseteq A$?

2. Is $B \subseteq D$?

3. Is $E \subseteq A$?

4. Is $C \subseteq E$?

5. Is $E \subseteq D$?

6. Is $2 \subseteq A$?

7. Is $B \subset E$?

8. Is $A \subset D$?

There are two other types of sets—or ways of categorizing sets. That is, **finite** and **infinite**.

A **finite** set has a whole number cardinality. Or, the cardinality of a finite set is a whole number.

An **infinite** set, then, does <u>not</u> have a whole number cardinality.

Another definition of an **infinite** set is a set which can be placed in a one-to-one correspondence with a proper subset of itself.

An infinite set is said to be **countably infinite** if it can be put in a one-to-one correspondence with the set of natural numbers.

Have we seen any such sets so far?

What are some others?

The set of **Real** numbers is an example of a set that is **uncountably infinite**….or **uncountable.** Why?