The Order of a Circuit: With the addition of capacitors and inductors to the list of ideal basic elements out of which our circuits can be constructed, the mathematical sophistication needed in order to analyze them has increased significantly. This is due to the fact that the iv-characteristics for both the capacitor and inductor are differential (or integral) equations, rather than algebraic equations, as is the case for the resistor (Ohm's law) and for the voltage and current sources, both independent and dependent. Circuit analysis is still possible with the use of Kirchhoff's laws along with the iv-characteristics of each ideal basic circuit element, but instead of setting up and solving systems of linear algebraic equations as was done before the addition of capacitors and inductors, now KVL and KCL will be used to set up systems of differential equations. It can be shown that the order of the differential equation that needs to be solved to analyze the circuit is equal to the number of energy storage elements, i.e. capacitors and inductors, present in the circuit. We begin with the analysis of first-order circuits, which means that each circuit will contain either one capacitor or one inductor.

1) What is meant by "the order of a differential equation"?

2) Give an example of a first-order differential equation.

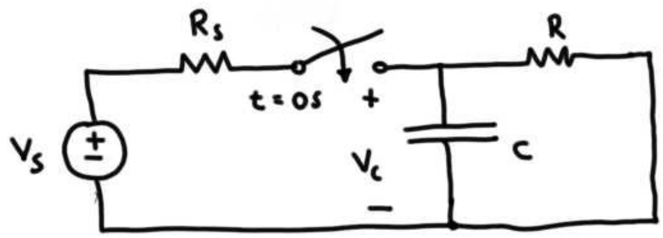
3) Give an example of a second-order equation.

4) Give an example of a fifth-order equation.

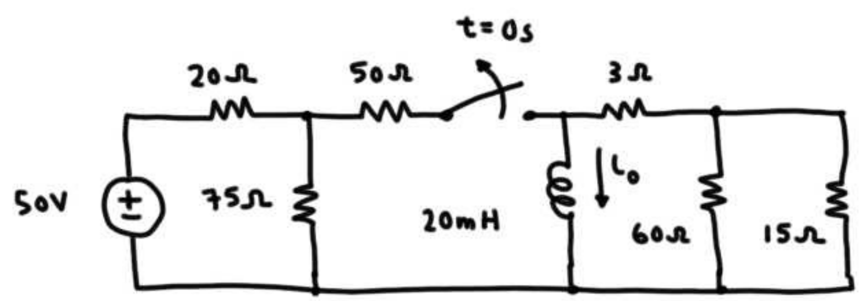
The examples can be as simple or as complicated as you wish. Also, clearly state which is the independent variable and which is the dependent variable in the differential equation.

Determining the Order: For each of the circuits shown below, determine the order of the differential equation that needs to be solved to analyze the circuit.

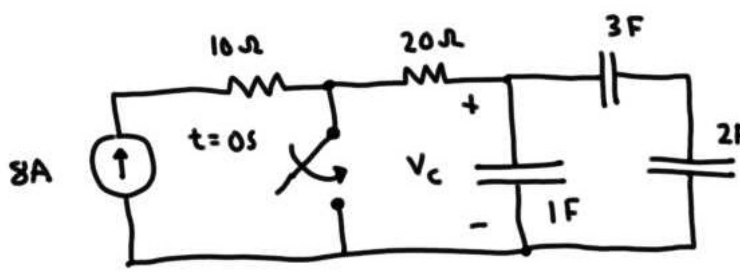
1) Answer:



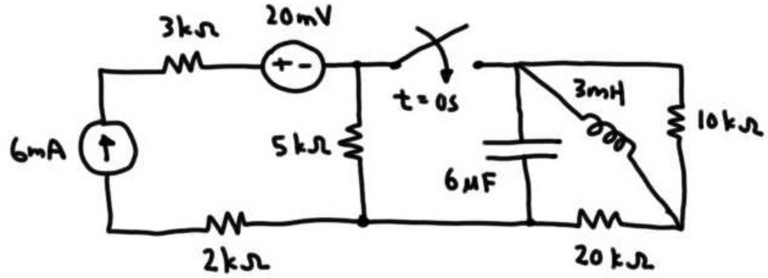
2) Answer:



3) Answer:



4) Answer:



The Response of a Circuit: Since the only sources that will be in the circuits will be DC sources, while the circuit element electrical parameters are time dependent, all our circuits must contain one or more switches to insure the circuits are not perpetually in DC steady-state.

1) What is the difference between the (a) natural response and (b) the step response of a circuit? Hint: think about what is happening to the energy storage element once the switch is flipped from its original position to its final position.

2) What is meant by “DC Steady-State”? What is a “transient”?

3) If a circuit is in DC steady-state then,
   a) how is a capacitor modelled?

   b) how is an inductor modelled?

Hint: the capacitor and inductor can be replaced with an equivalent element that can be determined by the iv-characteristics. This is very similar to zeroing out independent sources with superposition.

4) Recall fundamental properties of capacitors and inductors.
   a) What MUST be continuous for a capacitor?

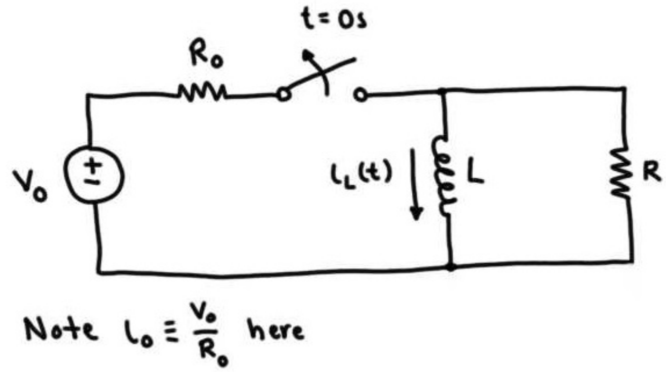
   b) What MUST be continuous for an inductor?

Note: These properties will be important when solving the ordinary-differential equations for the circuits.

The Natural Response of an RL Circuit:

Consider the circuit below:

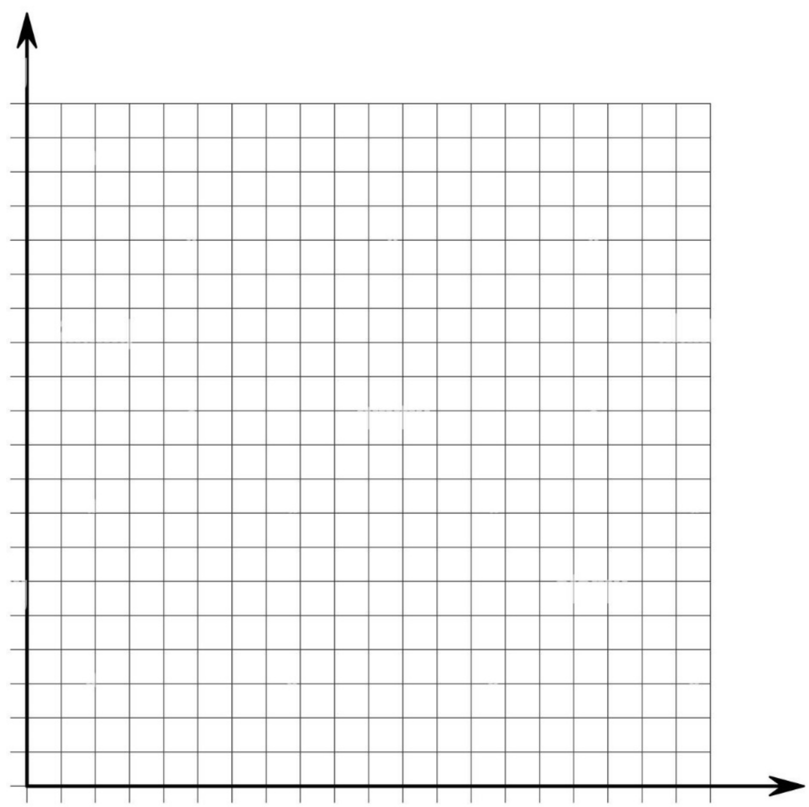
Given the variable definitions, set up and solve the differential equation for the current through the inductor, $i_L(t)$, for $t \geq 0\ s$. Express your solution in terms of i_0, L, R, t only.



Answer: $i_L(t) =$

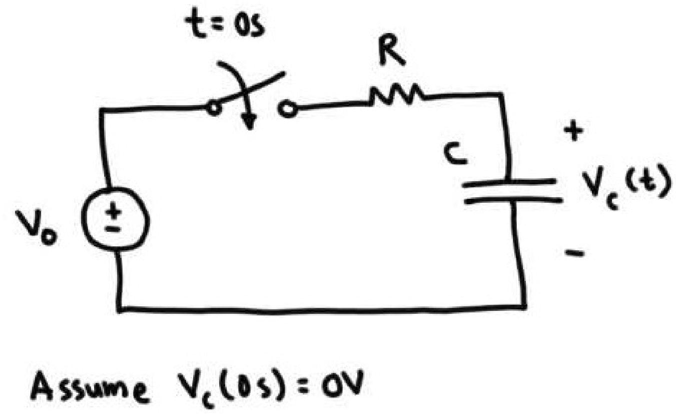
Graph how the inductor current varies as a function of time. Label your axes and choose the scale for both axes. This is meant to be more qualitative than quantitative.

$i_L(t)\ vs\ t$



The Step Response of an RC Circuit:

Consider the circuit below:

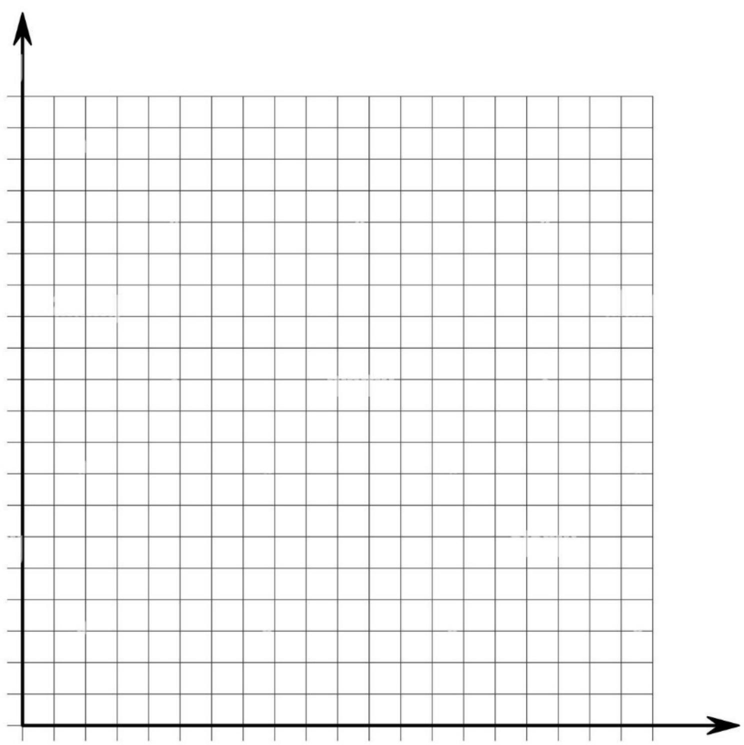


Given the variable definitions, set up and solve the differential equation for the voltage across the capacitor $v_C(t)$, for $t \geq 0\ s$. Express your solution in terms of V_0, C, R, t only.

<u>Answer:</u> $v_C(t) =$

Graph how the capacitor voltage varies as a function of time. Label your axes and choose the scale for both axes. This is meant to be more qualitative than quantitative.

$V_C(t)\ vs\ t$



Fundamental Properties of First-Order Circuits:

1) How is the time constant, τ, for an RC circuit calculated?

2) How is the time constant, τ, for an RL circuit calculated?

3) What is the physical meaning of the time constant? How does this affect how the circuit behaves?

    a) How does the circuit behave when the time constant is small?

    b) How does the circuit behave when the time constant is large?