Classwork   **Name** _____

Sections 6.1, 6.2 Factoring the GCF and factoring by grouping

Factor each completely.

1. $3x^4 + 12x^3 - 6x^2 + 3x$

7. $2x(5x+3) - 7(5x+3)$

2. $35y^3 - 21y$

8. $-c^3 - c$

3. $12a^3b^3 - 18a^2b + 6ab$

9. $-2y^3 - 8y^2 - 4y$

4. $-36w^3 - 9w$

10. $4y^2 - 9$

5. $3(x-1) + y(x-1)$

11. $50w^3 - 18w$

6. $-10x^5 + 15x^3 - 5x^2$

12. $y^2(k+5) - 16(k+5)$

Use grouping to factor each polynomial completely.

13. $10xy + 2y + 15x + 3$

18. $12a^2 - 9a + 20a - 15$

14. $w^3 - 3w^2 + 2w - 6$

19. $10k^4 - 2k^3 + 20k^2 - 4k$

15. $2py - qy - 4pw + 2qw$

20. $2 + w^2 - 2w^2 - w^4$

16. $xy + 2y - 3x - 6$

21. $12k^3 + 10k^2 - 6k - 5$

17. $9p^2 - 5pq - 18qp + 10q^2$

22. $9 - 9y^2 - y + y^3$

Classwork   **Name** _____
Section 6.2   Factoring Difference of Two Squares and Perfect Square Trinomials

Difference of Two Squares: $a^2 - b^2 = (a-b)(a+b)$
Perfect Square Trinomials: $a^2 + 2ab + b^2 = (a+b)^2$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Factor completely:
1) $y^2 - 49$ 6) $81w^4 - 1$

2) $9x^2 - 1$ 7) $9x^2 + 1$

3) $49x^2 - 25$ 8) $81w^4 - 16y^4$

4) $1 - 4y^2$ 9) $44w^3x^2 - 99w^3$

5) $2a^2 - 50$ 10) $9 - y^2$

16) $y^2 - 8y + 16$

11) $a^2 - b^2$

17) $4x^2 + 12x + 9$

12) $45y^2 - 245$

18) $x^2 + 10x + 25$

13) $121x^4 - 64y^4$

19) $9x^2 + 6x + 1$

14) $a^4 - 256$

20) $49x^2 + 28x + 4$

15) $x^2 + 6x + 9$

Factoring Trinomials Using Grouping

Example 1: **Factor** $3x^2 + 8x + 5$

1. Find the product of the coefficient of the first term and the last term:

$$3 \text{ times } 5 = 15$$

2. Make a table with all factor pairs of 15, beginning with 1 and 15.
    Find the sum of each factor pair. **The pair which works is 3 and 5 since their sum is 8 (the coefficient of the degree 1 term.)**

multiply to 15	add to 8
1, 15	1 + 15 = 16
3, 5	3 + 5 = 8

3. Write trinomial with four terms, splitting the middle term into the pair:

$$3x^2 + 8x + 5 \ = \ 3x^2 + 3x + 5x + 5$$

4. Break into pairs and remove greatest common factor from each pair:

$$\left(3x^2 + 3x\right) + \left(5x + 5\right) \ = \ 3x(x+1) + 5(x+1)$$

5. Factor out common binomial of $(x+1)$. **The trinomial is now factored.**

$$(x+1)(3x+5)$$

6. Check your answer by multiplying out:

$$(x+1)(3x+5) \ = \ 3x^2 + 5x + 3x + 5$$

Alternative Method Using Rectangles:

Put each term into a rectangle.

$3x^2$	$3x$
$5x$	5

From each row and each column, factor out greatest common factor**:**

$$\begin{array}{c} \quad\quad\quad x \quad + \quad 1 \\ 3x \\ + \\ 5 \end{array}$$

$3x^2$	$3x$
$5x$	5

Write trinomial in factored form: $(x+1)(3x+5)$

Example 2:   **Factor** $3x^2 - 2x - 8$

1.   Find the product of the coefficient of the first term and the last term:
                                3 times -8 = -24

2.   Make a table with all factor pairs of -24, beginning with 1 and -24.   Note that the "larger number" should be negative, since the sum needs to be -2.   Find the sum   of each factor pair. **The pair which works is 4 and -6 since their sum is -2 (the coefficient of the degree 1 term.)**

multiply to -24	add to -2
1, -24	**1+(-24) = -23**
2, -12	**2 + (-12) = -10**
3, -8	**3 + (-8) = -5**
4, -6	**4 + (-6) = -2**

3.   Write trinomial with four terms, splitting the middle term into the pair:

$$3x^2 - 2x - 8 \;=\; 3x^2 + 4x - 6x - 8$$

4.   Break into pairs and remove greatest common factor from each pair:

$$\left(3x^2 + 4x\right) + \left(-6x - 8\right) \;=\; x\left(3x + 4\right) - 2(3x + 4)$$

5.   Factor out common binomial of $\left(3x + 4\right)$. **The trinomial is now factored.**

$$\left(3x + 4\right)\left(x - 2\right)$$

6.   Check your answer by multiplying out:

$$\left(3x + 4\right)(x - 2) \;=\; 3x^2 - 6x + 4x - 8$$

Alternative Method Using Rectangles:

Put each term into a rectangle.

$3x^2$	4x
-6x	-8

From each row and each column, factor out greatest common factor:

	3x	+	4
x	$3x^2$		4x
+			
-2	-6x		-8

Write trinomial in factored form: $(3x + 4)(x - 2)$

Class work   **Name** _____
Sections 6.3, 6.4 Factoring Trinomials Using Grouping
Factor each trinomial using the grouping method.

1. $2x^2 + 11x + 12$

multiply to 24	add to 11
1, 24	$1 + 24 = 25$

4. $6x^2 - 17x + 12$

multiply to 72	add to -17
$-1, -72$	$-1 + (-72) = -73$

2. $12y^2 + 17y + 6$

multiply to 72	add to 17
1, 72	$1 + 72 = 73$

5. $10x^2 - 23x - 21$

multiply to -210	add to -23
$1, -210$	$1 + (-210) = -209$

3. $m^2 + 19m + 84$

multiply to 84	add to 19
1, 84	$1 + 84 = 85$

6. $5y^2 + 23y + 24$

multiply to 120	add to 23
1, 120	$1 + 120 = 121$

7. $7x^2 + 19x - 6$

multiply to -42	add to 19
$-1, 42$	$-1 + 42 = 41$

10. $5x^2 + 13x + 7$

multiply to 35	add to 13
$1, 35$	$1 + 35 = 36$

8. $9y^2 - 21y + 10$

multiply to 90	add to -21
$-1, -90$	$-1 + (-90) = -91$

11. $6y^2 + 7y - 5$

multiply to -30	add to 7
$-1, 30$	$-1 + 30 = 29$

9. $p^2 - 19p + 48$

multiply to 48	add to -19
$-1, -48$	$-1 + (-48) = -49$

12. $4x^2 + 8xy + 3y^2$

multiply to 12	add to 8
$1, 12$	$1 + 12 = 13$

Classwork   **Name** _____

Sections 6.3, 6.4 Factoring Polynomials

Factor the following <u>completely</u>:

1. $t^2 + 18t + 72$

2. $y^2 + 8y + 16$

3. $16a^2 - b^2$

4. $25 - x^2$

5. $x^2 - 10x + 25$

6. $a^2 - 6a + 5$

7. $2x^2 + 5x + 2$

8. $x^3 + 7x^2 + 10x$

9. $2y^2 - 3y + 1$

10. $y^2 - 64$

11. $a^2 - 8a + 7$

12. $r^2 s^2 - 144$

13. $3y^4 + 3y^2$

14. $2c^2 - 200$

15. $2x^2 + 7x + 6$

16. $2a^2 - 2b^2$

17. $3x^2 - 5x - 12$

18. $x^2 + 11x + 24$

19. $3y^2 + 8y + 5$

20. $7a^2 + 22a + 3$

21. $3x^2 + 8x + 4$

22. $3a^2 + 14a + 15$

23. $8m^2 - 10m + 3$

24. $2y^2 - 7y + 3$

25. $2h^2 - h - 3$

26. $3y^2 + 5y - 2$

27. $3k^2 + 7k - 6$

28. $3m^2 - 7m - 6$

29. $6p^2 - p - 2$

30. $4b^2 + 5b - 6$

31. $2a^2 + 3a - 14$

32. $2x^2 + 5x - 12$

33. $6t^2 + 5t - 6$

34. $7n^2 - 22n + 3$

35. $2y^2 - 5y + 3$

36. $3x^2 + 4x - 15$

37. $2q^2 - 9q - 18$

38. $6y^2 - 11y + 4$

39. $6m^2 + 19m + 10$

40. $4y^2 - 17y - 15$

41. $6x^2 - 19x - 11$

42. $10n^2 - 19n + 7$

43. $12r^2 - 13r + 3$

44. $6y^2 - 19y + 15$

45. $18c^2 + 41c - 10$

46. $10k^2 - 11k - 6$

47. $6s^2 + 7s - 20$

48. $2a^2 - 72$

49. $m^3 + 6m^2 + 9m$

50. $4a^3 - 36a$

51. $6r^2 + 13r + 6$

52. $6y^2 - 24x^2$

53. $3y^2 + 21y - 24$

54. $6x^2 + 27x - 15$

55. $20y^2 + 34y + 6$

56. $12c^2 + 10c - 42$

57. $43y^3 - 12y^2 + 8y$

Classwork   **Name** _____
Section 6.6 Solving Quadratic Equations by Factoring

Solve the following equations by factoring:

1. $x^2 - 8x + 12 = 0$ 2. $p^2 + 9p + 20 = 0$ 3. $x^2 - 100 = 0$

4. $3d^2 + 14d - 5 = 0$ 5. $m^2 + 13m = 0$ 6. $3a^2 - 7a = 0$

7. $a^3 - a^2 - 20a = 0$ 8. $5a^2 + 34a - 7 = 0$ 9. $3a^2 - 22a + 7 = 0$

10. $t^5 - 20t^3 + 64t = 0$ 11. $4a^5 - 13a^3 + 9a = 0$ 12. $2a^2 + 23a + 56 = 0$

13. $2t^2 - 7t - 15 = 0$

14. $x^2 - 81 = 0$

15. $4a^2 - 17a + 4 = 0$

16. $b^2 + 3b = 40$

17. $12m^2 + 25m + 12 = 0$

18. $18n^2 - 3n = 15$

19. $n^3 = 9n$

20. $35z^3 + 16z^2 = 12z$

21. $12r^2 + 14r = 6$

Classwork   **Name** _____
Section 6.6 Applications

Write a quadratic equation for each problem and solve the equation to find the solution to the problem.

1.   The diagonal of a rectangle is 17 feet. The length of the rectangle is one less than twice the width. Find the length, width, perimeter and area of the rectangle. Draw the rectangle and label the width, the length and the diagonal. Use the Pythagorean Theorem to write an equation. Solve the equation.

Width   _____

Length   _____

Perimeter   _____

Area   _____

2.   The width of a rectangle is two less than three times the length. The area of the rectangle is 176 square meters. Find the length, width and perimeter of the rectangle. Draw the rectangle and label the length and the width. Use the area of a rectangle to write an equation. Solve the equation.

Width   _____

Length   _____

Perimeter   _____

3.   The sum of squares of three consecutive odd integers is 371. Find the integers. Write and equation and solve it.

1st integer   _____

2nd integer   _____

3rd integer   _____

4.   The area of a triangle is 174 square inches. The base is five more than twice the height. Find the base and height of the triangle.   Draw the triangle and label the base and height. Use the area of a triangle to write an equation. Solve the equation.

Base   _____

Height   _____

Classwork   **Name** _____
Section 7.1 Reducing Rational Expressions

Simplify each algebraic function. Assume no denominator is equal to zero.

1. $\dfrac{5x^2 - 15x}{10x^2}$

2. $\dfrac{u^2 - u - 2}{u^2 + u}$

3. . $\dfrac{s^2 - t^2}{(t - s)^2}$

4. $\dfrac{x^2 - 5x + 6}{x^2 - 7x + 12}$

5. $\dfrac{x - 7}{7 - x}$

6. $\dfrac{x^2 + 2x - 8}{(2 - x)(4 + x)}$

7. $\dfrac{t^2 - 9}{t^3 - 9t}$

8. $\dfrac{6x^3 y}{6xy - 12x^4 y^2}$

9. $\dfrac{x^2 + 7x + 6}{x^2 - 1}$

10. $\dfrac{48a^2b^5c}{32a^7b^2c^3}$

11. $\dfrac{-28x^3y^4z^5}{42xyz^2}$

12. $\dfrac{k+3}{4k^2+7k-15}$

13. $\dfrac{2d^2+4d-6}{d^4-10d^2+9}$

14. $\dfrac{6g^2-19g+15}{12g^2-6g-18}$

15. $\dfrac{6y^2-5y+1}{1-y-6y^2}$

Classwork   **Name** _____
Section 7.2 Multiplying and Dividing Rational Expressions

Multiply and Divide the following:

1. $\dfrac{1}{4} \bullet \dfrac{4x}{1}$

2. $\dfrac{2}{3} \bullet \dfrac{9y^2}{4}$

3. $-\dfrac{6a^2}{5} \div \dfrac{3a}{2}$

4. $-\dfrac{7x}{4} \div x^3$

5. $\dfrac{-6ab}{3} \bullet \dfrac{4a}{8ab^2}$

6. $-\dfrac{24xy^2}{8x} \bullet \dfrac{21x^2y}{14y}$

7. $\dfrac{r^2 - rs}{rs} \bullet \dfrac{rs}{2r - 2s}$

8. $\dfrac{2a - 2b}{ab} \bullet \dfrac{ab}{4a - 4b}$

9. $\dfrac{3y - 9}{15 - 5y} \div \dfrac{8y - 4}{10y - 5}$

10. $\dfrac{4n-8}{3n} \div (6n-12)$

11. $(3x-3) \div \dfrac{x^2-1}{x}$

12. $\dfrac{n^2-3n-10}{n^2+2n-35} \bullet \dfrac{n^2+4n-21}{n^2+9n+14}$

13. $\dfrac{x^2-x-20}{x^2+7x+12} \div \dfrac{x^2-7x+10}{x^2+9x+18}$

Classwork   **Name** _____

Section 7.4 Adding and Subtracting Rational Expressions

Add and Subtract the following:

1. $\dfrac{x+1}{2a}+\dfrac{x-1}{2a}$

2. $\dfrac{3}{a+b}-\dfrac{a+3}{a+b}$

3. $\dfrac{3a+2b}{4a-2b}-\dfrac{a+2b}{4a-2b}$

4. $\dfrac{x}{x+1}+\dfrac{1}{x+1}$

5. $\dfrac{a}{a^2-b^2}-\dfrac{b}{a^2-b^2}$

6. $\dfrac{2}{x}+\dfrac{7}{y}$

7. $\dfrac{3}{ax}-\dfrac{5}{a^2x^2}$

8. $\dfrac{2a-b}{2b}+\dfrac{a+b}{a}$

9. $\dfrac{3x+2y}{3y}-\dfrac{x+2y}{6x}$

10. $\dfrac{7}{5a-10}+\dfrac{5}{3a-6}$

11. $\dfrac{r}{r+3}+\dfrac{r}{r-3}$

12. $\dfrac{5}{c+2}-\dfrac{3}{c-2}$

13. $\dfrac{7}{x-3}+\dfrac{3}{3-x}$

14. $\dfrac{x+4}{2x^2-2x}+\dfrac{5}{2x-2}$

15. $\dfrac{5}{x^2+5x}-\dfrac{10}{x^2-25}$

16. $x+\dfrac{2}{x-2}$

17. $x+1+\dfrac{2}{x-2}$

18. $\dfrac{x}{x^2-9}+\dfrac{x+1}{x^2-4x+3}$

Classwork   **Name** _____
Sections 7.6 and 7.7 Solving Rational Equations and Proportions
Solve the following.   Remember a denominator may not be zero.

1. $4 + \dfrac{9}{x^2} = \dfrac{12}{x}$

2. $6 + \dfrac{11}{y} + \dfrac{3}{y^2} = 0$

3. $\dfrac{2}{x+1} + \dfrac{1}{x+1} = 3$

4. $\dfrac{3}{z-2} + \dfrac{5z}{z+2} = 5$

5. $\dfrac{1}{t+4} + \dfrac{4t}{t-4} = 4$

6. $\dfrac{1}{z+2} + \dfrac{z}{z+6} = 1$

7. $(t+2)^2 - (t-5)^2 = 4$

8. $(2r+1)^2 - (r+1)^2 = 0$

Solve the following proportions:

9. $\dfrac{2}{x+1} = \dfrac{1}{x-2}$

10. $\dfrac{x}{7} = \dfrac{7}{x}$

11. $\dfrac{a-4}{a+5} = \dfrac{3}{8}$

12. $\dfrac{5}{x-1} = \dfrac{3}{x+2}$

13. $\dfrac{4}{t-5} = \dfrac{t-1}{t-5}$

14. $\dfrac{2}{t-9} = \dfrac{t-7}{t-9}$

15. $\dfrac{3}{x+4} = \dfrac{5}{x}$

16. $\dfrac{4}{8-a} = \dfrac{4-a}{a-8}$

17. $\dfrac{x^2}{x-1} = \dfrac{1}{x-1}$

18. $\dfrac{x-2}{x+1} = \dfrac{x-4}{x-6}$

LS 10A Review for Exam 3   **Name** _____

For Problems 1 – 20, factor each polynomial as much as possible.

1) $y^2 - 13y + 12$ 7) $4a^2 - y^2$

2) $m^2 - 4m - 45$ 8) $81a^4 - 1$

3) $x^2 + 18xy + 65y^2$ 9) $6m^3 + 15m^2 + 8m + 20$

4) $n^2 + 29n + 180$ 10) $10p^3 + 6p^2 - 5p - 3$

5) $25x^2 - 1$ 11) $3x^3 + 18x^2 + 27x$

6) $1 - b^2$ 12) $20y^2 - 45$

13) $10x^2 + 17x + 3$

14) $21x^2 + 41x + 10$

15) $2m^2 - m - 3$

16) $12y^2 + 8y - 15$

17) $-11y + y^2 - 42$

18) $9x^2 - 24x + 16$

19) $4w^2 + 20w + 25$

20) $5(x - 2y) - x(x - 2y)$

For Problems 21 – 30, perform the indicated operation. Write your answer in lowest terms.

21) $\dfrac{24a^3b^5}{-6a^2b}$

22) $\dfrac{49x^2 - 25}{42x + 30}$

23) $\dfrac{20y^3 + 15y^2 + 4y + 3}{4y^3 + 3y^2 + 8y + 6}$

- 67 -

24) $\dfrac{g^2+7g+10}{g^2+2g-15}\cdot\dfrac{4g-4}{3g+6}$

28) $\dfrac{5}{21x^2y}+\dfrac{6}{35xy^3}$

29) $\dfrac{12}{4x^2-9}+\dfrac{2}{2x+3}$

25) $\dfrac{2a^2-5a-12}{a^2-9a+18}\div\dfrac{4a^2-9}{a^2-10a+24}$

30) $\dfrac{x+1}{x^2-3x+2}+\dfrac{3}{x-2}$

26) $\dfrac{27w^4y}{26wy^4}\bullet\dfrac{65w^2y}{108w^5y^3}$

For Problems 31 – 44, solve each equation.

31) $(2x-1)(x+7)=0$

27) $\dfrac{12n}{3n-2}-\dfrac{8}{3n-2}$

32) $5y(y-9)=0$

37) $3x^2+19x+20=0$

33) $m^2-7m-18=0$

38) $10m^2+17m+3=0$

34) $y^2+7y=30$

39) $h(h-5)=-4$

35) $x^2+11x=0$

40) $n^2=100$

36) $9x^2=25$

41) $(x-5)(x+8)=48$

42) $\dfrac{4y+7}{5} = \dfrac{3y-11}{10}$

46) $(x+3)^2 - (x-6)^2 = 9$

47) $7 + \dfrac{22}{y} + \dfrac{3}{y^2} = 0$

43) $\dfrac{9-5x}{x} = \dfrac{4}{7}$

48) $\dfrac{8}{w-3} + \dfrac{6w}{w+3} = 6$

44) $\dfrac{m}{3m+4} = \dfrac{1}{m}$

49) $\dfrac{5}{9}x + \dfrac{7}{8}x = 3$

45) $\dfrac{9}{p} = \dfrac{p}{p-2}$

50) $\dfrac{3}{x} - 7 = \dfrac{4}{x}$

52) The length of a rectangle is three less than four times the width. The area of the rectangle is 175 square feet. Find the width, length and perimeter of the rectangle. Draw the rectangle and label the length and the width. Write an equation for the area of the rectangle. Solve the equation.

51) The diagonal of a rectangle is 15 feet. The length of the rectangle is three more than the width. Find the length, width, perimeter and area of the rectangle. Draw the rectangle and label the width, the length, and the diagonal. Use the Pythagorean Theorem to write and equation. Solve the equation.

Answers to Review for Exam 3

1) $(y-12)(y-1)$

2) $(m-9)(m+5)$

3) $(x+13y)(x+5y)$

4) $(n+20)(n+9)$

5) $(5x-1)(5x+1)$

6) $(1-b)(1+b)$

7) $(2a+y)(2a-y)$

8) $(3a+1)(3a-1)(9a^2+1)$

9) $(3m^2+4)(2m+5)$

10) $(2p^2-1)(5p+3)$

11) $3x(x+3)^2$

12) $5(2y+3)(2y-3)$

13) $(5x+1)(2x+3)$

14) $(3x+5)(7x+2)$

15) $(2m-3)(m+1)$

16) $(2y+3)(6y-5)$

17) $(y-14)(y+3)$

18) $(3x-4)^2$

19) $(2w+5)^2$

20) $(x-2y)(5-x)$

21) $-4ab^4$

22) $\dfrac{7x-5}{6}$

23) $\dfrac{5y^2+1}{y^2+2}$

24) $\dfrac{4(g-1)}{3(g-3)}$

25) $\dfrac{(a-4)^2}{(a-3)(2a-3)}$

26) $\dfrac{5}{8y^5}$

27) 4

28) $\dfrac{25y^2+18x}{105x^2y^3}$

29) $\dfrac{2}{2x-3}$

30) $\dfrac{2(2x-1)}{(x-1)(x-2)}$

31) $\dfrac{1}{2}$ or -7

32) 0 or 9

33) 9 or -2

34) -10 or 3

35) -11 or 0

36) $\dfrac{5}{3}$ or $-\dfrac{5}{3}$

37) $-\dfrac{4}{3}$ or -5

38) $-\dfrac{1}{5}$ or $-\dfrac{3}{2}$

39) 1 or 4

40) -10 or 10

41) -11 or 8

42) -5

43) $\dfrac{21}{13}$

44) -1 or 4

45) 3 or 6

46) 2

47) $-\dfrac{1}{7}$ or -3

48) $\dfrac{39}{5}$

49) $\dfrac{216}{103}$

50) $-\dfrac{1}{7}$

51) $w^2 + (w+3)^2 = 15^2$
        $width = 9 \; feet$
        $length = 12 \; feet$
        $Perimeter = 42 \; feet$
        $Area = 108 \; square \; feet$

52) $w(4w-3) = 175$
        $width = 7 \; feet$
        $length = 25 \; feet$
        $Perimeter = 64 \; feet$